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A COMPUTER PROGRAM TO CALCULATE INCOMPRESSIBLE LAMINAR AND TURBULENT BOUNDARY LAYER DEVELOPMENT

by H. J. Herring and G. L. Mellor

Prepared by
PRINCETON UNIVERSITY
Princeton, N. J.
for Lewis Research Center

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16. Abstract

Described herein is a computer program which performs a numerical integration of the equations of motion for an incompressible two-dimensional boundary layer. Boundary layer calculations may be carried out for both laminar and turbulent flow for arbitrary Reynolds number and mainstream velocity distribution, on planar or axisymmetric bodies with wall suction or blowing and with a rough or a smooth wall. A variety of options are available as initial conditions. The program can generate laminar initial conditions such as Falkner-Skan similarity solutions (so that initial wedge flows can be simulated including Blasius or stagnation point flow) or equilibrium turbulent profiles can be generated. Alternatively, initial profile input data can be utilized.

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FOREWORD

TABLE OF CONTENTS

The number in curved brackets following some of the notational definiations is the number of the equation in the text which defines the variable. The quantity in square brackets is the equivalent notation used in the computer program.

$A(\beta)$ Parameter in skin friction equation (23).

A_f Constant used to match numerical to asymptotic solution (38), [CAF].

$a_m^{(\ell)}$ Variables used in the Runge-Kutta method (35), [Aℓm], m = 1,4; ℓ = 1,3.

A_s Coefficient in asymptotic solution (39b), [ASF].

B Exponent in expression for freestream velocity variation in laminar similarity solution (20), [BS].

$b_1 .. b_5$ Coefficients in linearized momentum equation (32), [B1..B5].

B_s Parameter in skin friction equation (23).

C Constant of proportionality in expression for δ^* in laminar similarity solution (20), [C].

C_a $= 2(\delta^*/r_w) \cos \alpha$, parameter related to axisymmetric flow appearing in equation (11), [CA].

C_f $= \tau_w/\rho U^2/2)$, coefficient of skin friction, [CF].

c_s Parameter in equation (16), [SC].

C_w Radius of longitudinal curvature, [CW].

$c_1 .. c_4$ Coefficients in equation (30), [C1..C4].

$f^{(k)}$ Variables used in the Runge-Kutta method (35), k = 1,3.

f' $= (U-u)/U$, velocity defect variable, [FP].

f'_p Particular solution of equation (32), [FP].

f'_h Homogeneous solution of equation (32), [VHP].

H $= \delta^*/\theta$, shape factor, [SF].

i,j Indices of variables in the x and y directions respectively, [I,J].

L = $x_2 - x_1$, position at which R_L is defined in laminar similarity (19).

P = $\delta^* U_x / U$, parameter in equation (11), [P].

P^* = $R_{\delta^*} P$, parameter in laminar similarity flow, [P].

Q = $(U\delta^*)_x / U$, parameter in equation (11), [Q].

Q^* = $R_\delta * Q$, parameter in laminar similarity flow, [Q].

R = $r_{w_x} \delta^* / r_w$.

R^* = $R_{\delta^*} R$, parameter in laminar similarity flow, [R].

r_w Radius of surface in same units as x, [RW].

R_L = $(x_2 - x_1) U / \upsilon$, Reynolds number in laminar similarity solution, [RL].

R_{δ^*} = $\delta^* U / \upsilon$, Reynolds number based on displacement thickness, [RDT].

r = $r_w(\bar{x}) + \bar{y} \cos \alpha(\bar{x})$, radius of a point in the boundary layer in same units as x.

s_w Characteristic size of roughness elements in same units as x, [SW].

s Parameter in asymptotic outer solution (16), [S].

T Proportion of turbulence viscosity in effective viscosity (43), [TURB].

u, v Time average velocities in the x and y directions respectively.

U Free stream velocity; arbitrary dimensional units, [U].

U_L Free stream velocity at x_2 in laminar similarity solution in same units as U, [U(2)].

$\overline{-u'v'}$ Reynolds stress

u_τ = $\sqrt{\tau_w / \rho}$, skin friction velocity in same units as U.

v_w Wall transpiration velocity in same units as U, [VW].

x Streamwise coordinate; arbitrary dimensional units, [X].

y Coordinate normal to wall.

Δx = $x_{i+1} - x_i$, numerical integration step in the streamwise direction, [DX].

α Angle of the tangent to the surface with respect to the axis of symmetry.

β $\delta^*(dp/dx)/\tau_w$, the Clauser equilibrium pressure gradient parameter, [B].

γ $= \sqrt{\tau_w/\rho U^2}$, ratio of skin friction velocity to free stream velocity, [GAM].

δ^* $= \int_0^\infty (\bar{U}-\bar{u})/\bar{U}(r/r_w)d\bar{y}$, displacement thickness in same units as $\bar{x}$, [DT].

η $= y/\delta^*$, nondimensional coordinate normal to wall, [Y].

θ $= \int_0^\infty \bar{u}(\bar{U}-\bar{u})/\bar{U}^2 (r/r_w)d\bar{y}$, momentum thickness in same units as $\bar{x}$, [MT].

κ $= 0.41$, von Karman constant, [SK].

υ Molecular kinematic viscosity.

υ_e Effective kinematic viscosity, (4).

ρ Density.

$\bar{\tau}$ Local shear stress, (4).

$\bar{\tau}_b'$ $= \partial(r\bar{\tau}/r_w\rho U^2)/\partial\eta$, nondimensional shear stress gradient, (30), [TPB].

T Nondimensional effective viscosity, (12), [VE].

T^* $= R_\delta * T$, nondimensional effective viscosity in laminar starting flow, [VE].

φ, Φ Inner and outer effective viscosity functions shown in Figure (2).

X $= \kappa y\sqrt{\bar{\tau}/\rho}/\delta^* U$, coordinate normal to wall in outer effective viscosity hypothesis, [CHI].

χ $= \kappa y\sqrt{\bar{\tau}/\rho}/\upsilon$, coordinate normal to wall in inner effective viscosity hypothesis.

Superscripts and Subscripts

$( )_a$ Evaluated at asymptotic matching point, [( )A].

$( )_s$ Evaluated at point where recalculation begins.

$( )_w$ Evaluated at wall, [( )W].

$( )_x$ Differentiation with respect to x.

$( )_1$ Evaluated at initial x station.

$( )_\infty$ Evaluated at the edge of the layer, $\eta \to \infty$, $[( )E]$.

$( )'$ Differentiation with respect to $\eta = y/\delta^*$, $[( )P]$.

$\overline{(\ )}$ Used with $\bar{u}, \bar{v}$, etc. denotes untransformed coordinates. Used with functions of x only, denotes average value, $[( )_{i+1} + ( )_i]/2$.

$\underset{\smile}{(\ )}$ Denotes quantity in similarity starting equation (32) which has different interpretation in laminar and turbulent flows.

$\$( )$ Denotes subroutine of program.

I. INTRODUCTION

Studies of boundary layer flows have been made for two reasons; one is the practical need for boundary layer solutions in design problems; the other is the desire to achieve a better theoretical understanding of the mechanism of boundary layer flows. The calculation method described herein was designed to expedite both of these objectives. It makes recent advances in the state of the art available in the form of a convenient tool for those who are interested in ends rather than means. For those concerned with theoretical investigations of boundary layer flows, it overcomes the technical problems of solving the equations of motion and thereby emphasizes the physical assumptions necessary to circumvent our ignorance and inability to describe basic turbulent transport processes. A variety of assumptions can, therefore, be tested free from approximations related to the solution of the equations. Although a specific turbulent effective viscosity hypothesis is included in the program for practical calculations, it is wholly contained in a subroutine. The subroutine may easily be replaced by an alternative form.

Somewhat more involved extensions of this program have, in fact, been used to investigate more complicated models which calculate mean turbulent energy fields and, at the Stanford Conference on Computation of Turbulent Boundary Layers [1], have been compared with calculations using the more simple effective viscosity hypothesis. Nevertheless, this simple hypothesis performs remarkably well in predicting data and this has now been well documented in the literature [2,3,4]. Therefore, it is possible to concentrate on computational details in this report. Also, of course, the program can be operated entirely in a laminar mode where the problem is purely numerical.

Various versions of the program have now been in existence quite a number of years. However, it is one matter to have a program that works, but it is another matter to publish a program for general consumption and to provide sufficient (though not exhaustive) documentation. Furthermore, considerable effort has now been expended to enable the program to handle flows of wide generality while avoiding numerical trauma.

Aside from the capability to compute planar or axisymmetric, laminar or turbulent flows with arbitrary pressure gradients and Reynolds number, provision has been made to calculate flows with wall transpiration or aspiration and wall roughness. In these latter cases, predictability of data has not yet been documented in the literature; however, informal comparisons have been favorable. Internal means to effect transition from turbulent flow have not been provided; rather a transition factor (TURB), which varies from zero for laminar flow to unity for fully turbulent flow, must be provided by the user as input. Undoubtedly, existing transition data could be incorporated in the program on a purely empirical basis, or, hopefully, a meaningful semi-empirical model of transition will be constructed in the future. Finally, the user will notice that there

are input and output statements containing a longitudinal wall curvature variable which, as yet, plays no role in the calculation. Experimental research on this effort (see reference to the effect of curvature in [4] and more recently some suggestions on ways of including it in [5]) is in progress at Princeton.

This manual is intended to be more than a catalog of the inputs and outputs. The boundary layer equations of motion are traced up to the point of being recast for computation and some discussion of the philosophy of choice of numerical methods is included. The alternative of reading in an initial profile or internally generating a laminar similarity (Falkner-Skan) or turbulent similarity (equilibrium) profile is discussed. (It should be noted that the main program routine is fairly long due to the number of initialization options which are provided. The principal forward streamwise calculations begin after statement 480; the reader seeking detailed understanding of the main program routine may be advised to start at this point.) Then a step-by-step description of the function of each section of the program is given along with a flow chart and a list of notations. Finally, the practical problem of setting up the input parameters to calculate a specific flow is considered. Users who are not interested in the theoretical basis of the calculation method may skip directly to Section V, the description of inputs and outputs. Then if specific questions arise, reference can be made to earlier sections.

The basic numerical scheme can be described as an implicit, Crank-Nicholson scheme resulting at each station in an ordinary differential equation which is solved according to a Runge-Kutta method adapted to the laminar and turbulent boundary layer equations. The ordinary differential equation is solved by a linearization-iteration technique which apparently (compare [6] and [7]) is essential to the solution of turbulent boundary layers. Another useful feature of the program is that the normal coordinate is scaled on the displacement thickness so that while the displacement thickness varies in the stream direction, the distribution of the scaled normal coordinate can be fixed. (This may be modified if the program is applied to wall jets, for example, where the usual definition of displacement thickness may yield numbers that are near zero or negative. The modification is simple when it is realized the scaling variable need not be displacement thickness but any quantity chosen so that the region where the velocity gradient is nonzero is spanned by the scaled normal coordinate.) Furthermore, the velocity profile is normalized with the free stream velocity so that both variables,

$$f'(\eta, x) = \frac{U(x) - u(y, x)}{U(x)} \quad \text{and} \quad \eta = \frac{y}{\delta^*(x)}$$

do not vary greatly with x. Under these circumstances it is found that fairly large x-steps are possible. In this case small errors in δ^* (or θ) may be accumulated. However, this source of error may be significantly reduced by correcting δ^* according to a very accurate integral of the von Karman equation.

II. ANALYTICAL FUNDAMENTALS

Equations of Motion

The equations governing the flow of an incompressible, two-dimensional, boundary layer illustrated in Figure 1, are given by

$$\frac{\partial r\bar{u}}{\partial \bar{x}} + \frac{\partial r\bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \bar{U}\frac{d\bar{U}}{d\bar{x}} + \frac{1}{r}\frac{\partial [r(\bar{\tau}/\rho)]}{\partial \bar{y}} \tag{2}$$

where $r(\bar{x}, \bar{y}) = r_w(\bar{x}) + \bar{y}\cos\alpha(\bar{x})$. The equations apply to laminar or turbulent flow if the definition of $\bar{\tau}/\rho$ is taken to be

$$\bar{\tau}/\rho = \nu\frac{\partial \bar{u}}{\partial \bar{y}} - \overline{u'v'}, \tag{3}$$

where $-\overline{u'v'}$ is the kinematic Reynolds stress. We next define an effective viscosity so that

$$\bar{\tau}/\rho = \nu_e(\partial \bar{u}/\partial \bar{y}) \tag{4}$$

For laminar flow $\nu_e = \nu$.

The boundary conditions are

$$\bar{u}(\bar{x},0) = 0, \tag{5a}$$

$$\bar{v}(\bar{x},0) = \bar{v}_w(\bar{x}) \tag{5b}$$

$$\lim_{\bar{y}\to\infty} \int_0^{\bar{y}} [\bar{U}(\bar{x}) - \bar{u}(\bar{x},\bar{y}')]\left(\frac{r}{r_w}\right) d\bar{y}' \text{ is bounded.} \tag{5c}$$

First it is useful to transform equations (1,2) so that they appear closer to their planar form. This can be accomplished with a variation of the Probstein-Elliott transformation, [8]. Thus,
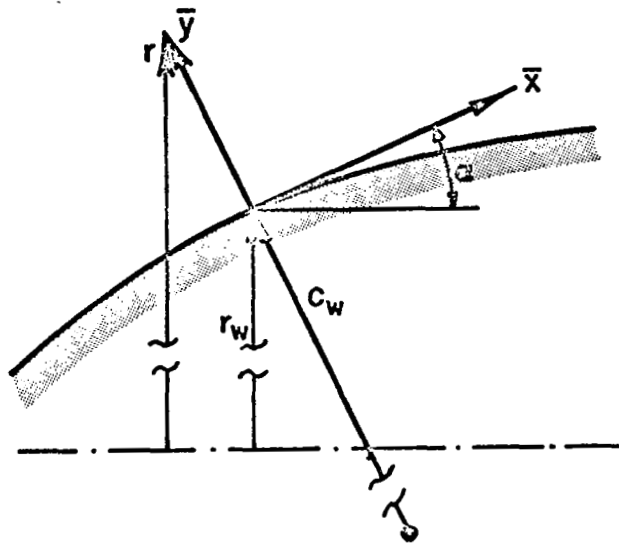
a.) Coordinate System
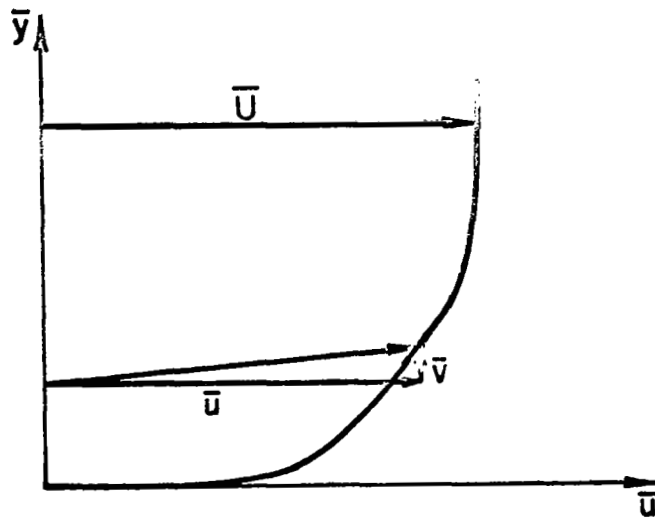
b.) Description of Velocity Profile

Figure 1. Illustration of Notation

$$x = \bar{x} \tag{6a}$$

$$y = \int_0^{\bar{y}} r(\bar{x}, \bar{y}')/r_w \, d\bar{y}' \tag{6b}$$

$$u(x,y) = \bar{u}(\bar{x}, \bar{y}) \tag{6c}$$

Here we write u without overbars even though, in the case of turbulent flow, it should be interpreted as a time averaged (or ensemble averaged) quantity. Using this transformation and the resulting relations,

$$\frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial x} + \frac{\partial y}{\partial \bar{x}} \frac{\partial}{\partial y} \quad \text{and} \quad \frac{\partial}{\partial \bar{y}} = \frac{r}{r_w} \frac{\partial}{\partial y}$$

equations (1) and (2) become

$$\frac{1}{r_w} \frac{\partial u \, r_w}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \frac{\partial}{\partial y} \left(\frac{r}{r_w} \frac{\bar{\tau}}{\rho} \right) \tag{8}$$

where now $\bar{\tau}/\rho = (r/r_w)\nu_e(\partial u/\partial y)$, $v = (r/r_w)\bar{v} + y_{\bar{x}} \bar{u}$, and $r^2 = r_w^2(x) + 2y\, r_w(x) \cos \alpha(x)$. The form of the boundary conditions is unchanged.

$$u(x,o) = 0, \tag{9a}$$

$$v(x,o) = v_w(x), \tag{9b}$$

$$\lim_{y \to \infty} \int_0^y [U(x) - u(x,y')]dy' \text{ is bounded.} \tag{9c}$$

For purposes of calculation it is convenient to define a new set of variables. The velocity profile is expressed in defect form

$$f'(x,\eta) = \frac{U(x) - u(x,y)}{U(x)} \; ; \quad \eta = \frac{y}{\delta^*(x)} \tag{10a,b}$$

The choice of (10a) is made because the calculation method is historically oriented toward turbulent flows in which case a defect formulation is convenient. Also some convenience results when considering outer boundary conditions. The coordinate, y, is normalized by $\delta^*(x) = \int_0^\infty (U-u)/U \, dy$. The more conventional scaling for laminar flows would be $\sqrt{U/\bar{v}x}$, but this is not meaningful in the turbulent case. Still no generality is lost since δ^* will be proportional to $\sqrt{U/vx}$ in the important Falkner-Skan laminar similarity flows. Finally, with an eye toward turbulent flows, the effective viscosity, v_e, is normalized on $U\delta^*$ so that $T = v_e/U\delta^*$; in turbulent flows T is a prescribed function of the local velocity profile whereas in laminar flows it is the inverse Reynolds number $v/U\delta^*$.

When rewritten in terms of these variables, equations (7) and (8) become

$$[(1 + C_a\eta)Tf'']' + [(Q + R)(\eta - f) - v_w/U]f'' + P(f'-2)f'$$

$$= \delta^*(1-f') \frac{\partial f'}{\partial x} + \delta^* f'' \frac{\partial f}{\partial x}. \tag{11}$$

where, $P = \delta^* U_x/U$, $Q = (\delta^* U)_x/U$, $R = \delta^* r_{w_x}/r_w$ and $C_a = 2(\delta^*/r_w) \cos \alpha$.

The form of the function T as given in Reference [2] is

$$T = \frac{1}{R_\delta^*} \phi(XR_\delta^*) + \Phi(X) - X, \quad X = \frac{Ky\sqrt{\tau/\rho}}{\delta^* U} . \tag{12a,b}$$

where ϕ and Φ are defined in Figure 2. The boundary conditions are

$$f'(x,o) = 1 \tag{13a}$$

$$f(x,o) = 0, \tag{13b}$$

$$\lim_{\eta \to \infty} f(\eta) \to 1. ** \tag{13c}$$

** The primary requirement is that $f(\eta)$ be bounded as $\eta \to \infty$. However, so long as $\eta = y/\delta^*$ the limit value is unity. On the other hand, throughout this text, δ^* could be considered simply as an arbitrary scale length, where $f(\infty)$ would assume the requisite value such that $f(\infty)\delta^*$ is the real displacement thickness.

$$\phi = 1 + \frac{\chi^4}{\chi^3 + (6.9)^3}$$

$$\chi = 0.41 \frac{y\sqrt{\bar{\tau}/\rho}}{\nu}$$
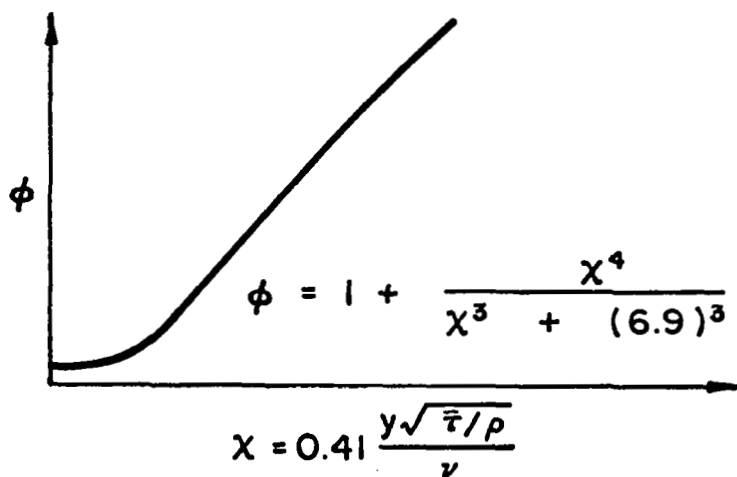
a.) The Inner Function, $\varphi = \varphi(\chi)$

$$\Phi = .016$$

$$\Phi = \chi$$

$$\chi = 0.41 \frac{y\sqrt{\bar{\tau}/\rho}}{U\delta^*}$$
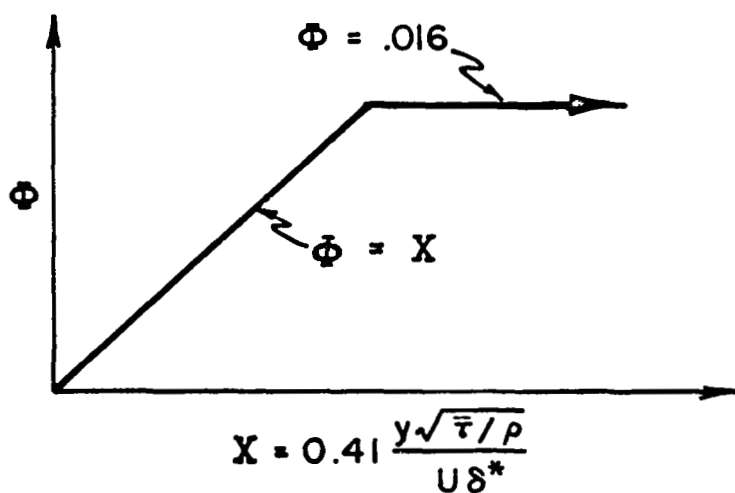
b.) The Outer Function, $\Phi = \Phi(\chi)$

Figure 2. The Turbulent Effective Viscosity Hypothesis

- 7 -

In the numerical solution of (11), (13c) must evidently be satisfied at some large but finite value of η. An improved outer boundary condition can be obtained from the equation (11). For large η,

$$[(1 + c_a\eta)\ T_\infty f'']' + (Q + R)\ (\eta - 1)\ f'' - 2Pf' = \delta* \frac{\partial f'}{\partial x}. \qquad (14)$$

Following the method indicated in Reference [4], a solution of (14) which is sufficiently accurate for these purposes can be written

$$f'(\eta) = f'(\eta_a)\ \exp\left[\frac{(\eta_a-1)^2 - (\eta-1)^2}{2s(x)\ (1+c_a\eta)}\right], \qquad (15)$$

where $s(x)$ is a solution of the equation

$$(\delta*/2)\ \frac{ds}{dx} + c_s\ (Q + R)s = T_\infty\ c_s^2, \qquad (16)$$

c_s is 1.0 for a two-dimensional flow and 0.5 for axisymmetric flow, and η_a is the value of η at which the numerical solution is matched to the asymptotic solution. Therefore equation (15), where s is obtained from (16), replaces (13c) as the outer boundary condition.

The complete set of equations described above is sufficient to calculate the development of a boundary layer. However, small inaccuracies in the numerical solution which are negligible after one step in x, are frequently cumulative. Greater accuracy can be achieved simply by correcting the integral parameters such as $\delta*$ and θ, obtained from the numerical solution of equation (11), by referring to a numerically more accurate solution of the same equation. This form is obtained by first integrating equation (11) across the layer which yields the von Karman integral momentum equation.

$$\frac{d}{dx}\ (r_w U^2\theta) + r_w \delta* U\frac{dU}{dx} = r_w\ \frac{\tau_w}{\rho} + r_w v_w U. \qquad (17)$$

Then, as suggested by Coles [9], equation (17) may be integrated with respect to x between two x stations x_{i-1} and x_i. The result is

$$\frac{\theta_i U_i^{2+\bar{H}} r_{w_i}}{\theta_{i-1}\ U_{i-1}^{2+\bar{H}}\ r_{w_{i-1}}} = \exp\left[\int_{x_{i-1}}^{x_i} \left(\frac{\overline{c_f}}{2} + \overline{\frac{v_w}{U}}\right)\ d\left(\frac{x}{\theta}\right)\right] \qquad (18)$$

where $\overline{H}$, $\overline{c_f}$, $\overline{\theta}$ and $(\overline{v_w}/\overline{U})$ are average values in the interval (x_{i-1}, x_i). Therefore, the left side, CO1, and right side, CO2, of (18) are first calculated and the calculated value θ_i is corrected by multiplication with the ratio CO2/CO1. As an indicator of the level of correction that has occurred, both CO2 and CO1 are printed output and a running product cumulative of CO2/CO1 (see definition, CMT in Section IV) is also printed.

Furthermore, if the finite difference solution were completely consistent the left-hand side of equation (18), as evaluated from the results at any two stations i-1 and i, and the right-hand side, obtained from the average skin friction coefficient and transpiration rate in effect between i-1 and i would be equal. The amount of imbalance is a check on the accuracy of the finite difference solution.

<div align="center">Initialization</div>

In order to compute a solution of equation (11), it is necessary to prescribe the velocity profile at the first x station. Although one may read in the profile as input data, often this is not readily known. What is likely to be known is the nature of the conditions under which the boundary layer developed in some region before the initial x station. For instance, it might be that the flow near x_1 is characterized by a member of the Falkner-Skan family of laminar wedge flows including the flat plate and stagnation point flow. Alternatively, in the case of turbulent boundary layer, it might be that the pressure gradient in the neighborhood of x_1 could be closely approximated by an equilibrium pressure gradient of the form proposed by Clauser [10]. In either case it is possible to generate profiles internally. The calculation of these similar solutions requires a relatively simple specialization of equation (11).

Laminar similarity flow

To begin with, in the laminar, Falkner-Skan case, it is convenient to multiply the similarity version of equation (11) by $R_{\delta*}$. Then setting $f'_x = f_x = 0$ the result is,

$$[(1 + C_a \eta)T*f'']' + [(Q* + R*)(\eta - f) - (\tfrac{v_w}{U} R_{\delta*})]f''$$

$$+ P*(f' - 2)f' = 0 \tag{19}$$

where $P* = \delta*^2 U_x/\nu$, $Q* = \delta*(\delta*U)_x/\nu$, $R* = \delta*^2 Ur_{w_x}/(\nu r_w)$, $T* = TR_{\delta*} = 1.0$

and C_a is the value of $2(\delta^*/r_w) \cos \alpha$ at x_2.**

The parameters P* and Q* may be specified in two ways. One is by making use of the fact that the mainstream velocity distribution, $U(x)$, and displacement thickness $\delta^*(x)$, are known (see [11], p.143) to be of the form

$$\frac{U(x)}{U_L} = \left(\frac{x}{L}\right)^B \quad \text{and} \quad \frac{\delta^*(x)}{L} = C\left(\frac{x}{L}\right)^{\frac{1-B}{2}} \quad \text{respectively.}$$

A value of $B = 0$ in the relations above corresponds to Blasius flow and $B = 1$ corresponds to stagnation flow. Intermediate values of B represent wedge included angles of $2\pi B/(B+1)$ (radians). For cones B may also be related to the cone included angle; see, for example, Reference [12] page 428. The parameters needed to solve equation (19) can easily be shown to be

$$P^* = C^2 R_L B,$$

$$Q^* = C^2 R_L (B+1)/2, \qquad \qquad (20a,b,c)$$

$$R^* = C^2 R_L L \; r_{w_x}/r_w$$

where $R_L = U L/\nu$ and C is determined in the course of solution so that $f(\infty) = 1$. Thus, R_L, B, and $L r_{w_x}/r_w$ specify the starting condition.

The other method of determining P*, Q*, and R* is to specify P*, $U\delta^*/\nu$, and $(v_w/U)R_{\delta^*}$. Then from the integral of equation (19),

$$(Q^* + R^*) = H[(C_f/2 + v_w/U)R_{\delta^*} - P^*] - P^*. \qquad (21)$$

Here the shape factor H, the skin friction coefficient, C_f, and therefore $(Q^* + R^*)$ are evaluated iteratively in the course of obtaining a solution, as described in Section III. From there, Q* is simply $(Q^* + R^*) - R^*$.

** With the exception of the limit case of axisymmetric stagnation point flow, exact similarity solutions do not exist at the vertex of cones; where δ^*/r_w is singular, the extent of the excluded region increases as the angle of the cone decreases. However, the procedure recommended here is probably sufficient to give good accuracy downstream of the vertex.

Turbulent similarity flow

In the case of equilibrium turbulent flow, it is known that $(U-u)/u_\tau \equiv f'_e \, (y\gamma/\delta*)$ is invariant with x, in which case (see Reference [2]),

$$\delta* f_x = \frac{\delta*\gamma_x}{\gamma} \, \eta f'$$

$$\delta* f'_x = \frac{\delta*\gamma_x}{\gamma} \, (\eta f'' + f').$$ (22a,b)

The factor, $\delta*\gamma_x/\gamma$, can be obtained from the skin friction equation for equilibrium flows (see Reference [3]).

$$\frac{1}{\gamma} = \frac{1}{\kappa} \, \mathrm{LOG}_e \, \frac{\delta* U}{\nu} \, + A(\beta) + B_s$$ (23)

which yields

$$\frac{\delta*\gamma_x}{\gamma} = - \frac{\gamma}{\kappa} \, \frac{(U\delta*)_x}{U} \, = - \frac{\gamma}{\kappa} \, Q.$$ (24)

Therefore, equation (11) may be rewritten in the form

$$[(1 + C_a\eta)Tf'']' + \left\{ (Q + R)[\eta(1 + \gamma/\kappa) - f] - (\gamma/\kappa)R\eta - \frac{v_w}{U} \right\} f''$$

$$+ [P(f' - 2) + (\gamma/\kappa)Q(1 - f')]f' = 0.$$ (25)

It still remains to specify the parameters P and Q. This can be done in two ways. One way is to prescribe $\beta = [(\delta*/\tau_w) \, (dp/dx)]$,

$$P = - (c_f/2)\beta$$ (26)

and Q can be obtained using the integral of equation (25) so that

$$Q = \frac{H[c_f/2 + (v_w/U) - P] - P - R}{1 + (\gamma/\kappa) (H - 1)}$$ (27)

where once again the shape factor and skin friction are evaluated iteratively in the course of obtaining the numerical solution for this profile.

The method above is quite satisfactory except for initial equilibrium boundary layers near separation. In that case, another approach is necessary since $\beta \to \infty$ and $C_f \to 0$, making equation (26) impractical for numerical computation. However, as shown in Reference [3], P is well behaved near separation and approaches the limit $P = -0.00948$ as $\beta \to \infty$. This limit is virtually independent of Reynolds number. Therefore, for large values of β, if P is fixed and Q is determined from equation (27), the solution converges rapidly.

Finally, the asymptotic form of equations (19) and (25) for large η is

$$[(1 + c_a \eta) \underset{\sim}{T}_\infty f'']' + [\underset{\sim}{Q}(1 + \underset{\sim}{\gamma}/\kappa) + \underset{\sim}{R}] (\eta - 1)f'' + (\underset{\sim}{Q}\gamma/\kappa - 2\underset{\sim}{P})f' = 0.$$

For laminar flow $\underset{\sim}{P} = P^*$, $\underset{\sim}{Q} = Q^*$, $\underset{\sim}{R} = R^*$, and $\underset{\sim}{T}_\infty = T_\infty^*$, whereas in turbulent flow $\underset{\sim}{P} = P$, $\underset{\sim}{Q} = Q$, $\underset{\sim}{R} = R$, and $\underset{\sim}{T}_\infty = T_\infty$. Also in laminar flow $\underset{\sim}{\gamma} = 0$ while in turbulent flow $\underset{\sim}{\gamma} = \sqrt{c_f/2}$. Assuming f' to have the form given in equation (15), s can be shown to be

$$s = \frac{\underset{\sim}{T}_\infty c_s}{\underset{\sim}{Q}(1 + \underset{\sim}{\gamma}/\kappa) + \underset{\sim}{R}}. \tag{28}$$

III. NUMERICAL METHOD

Equation (11), which describes the boundary layer flow, is a non-linear partial differential equation, parabolic in the flow direction. There are two phases of the procedure of obtaining a solution. The first phase is the conversion to an ordinary differential equation using finite differences for the x derivatives. The second phase is the method or solution of the resulting ordinary differential equation.

Reduction to Ordinary Differential Equation

In the first phase, the x derivatives are represented by finite differences in the x direction according to an adaptation of the Crank-Nicholson [13] scheme. This method is of the implicit type. It is always stable and the error is of second order in the x-step size.

The development of the difference equation is most clearly portrayed in three steps. The first step is to write equation (11) in terms of average functions at a point halfway between the x position of the known profile, x_{i-1}, and that of the profile to be calculated, x_i,

$$\overline{[(1 + C_a \eta)Tf'']}' + [(\overline{\delta^*_x} + \bar{P} + \bar{R})(\eta - \bar{f}) - \overline{\frac{v_w}{U}}]\bar{f}''$$

$$+ [\bar{P}(\bar{f}' - 2)]\bar{f}' = \overline{\frac{\delta^*}{\Delta x}}(1 - \bar{f}')(f'_i - f'_{i-1}) + \overline{\frac{\delta^*}{\Delta x}}\bar{f}''(f_i - f_{i-1}) \tag{29}$$

Then, using the relations

$$\bar{f}' = \tfrac{1}{2}(f'_i + f'_{i-1}), \text{ etc.}$$

Equation (29) can be written in terms of functions at position x_i as follows

$$-[(1 + C_a \eta)f'']'_i = -\tau'_b + c_1(f''_i + f''_{i-1}) + c_2(f'_i + f'_{i-1})$$

$$- c_3(f'_i - f'_{i-1}) - c_4(f_i - f_{i-1}) \tag{30}$$

- 13 -

where

$$c_1 = (\overline{\delta_x^*} + \bar{P} + \bar{R}) \, [\eta - \tfrac{1}{2}(f_i + f_{i-1})] - (v_{w_i} + v_{w_{i-1}})/(U_i + U_{i-1}),$$

$$c_2 = \bar{P}[\, \tfrac{1}{2}(f'_i + f'_{i-1}) - 2],$$

$$c_3 = (\delta_i^* + \delta_{i-1}^*)[1 - \tfrac{1}{2}(f'_i + f'_{i-1})]/\Delta x$$

$$c_4 = (\delta_i^* + \delta_{i-1}^*) \; \tfrac{1}{2}(f''_i + f''_{i-1})/\Delta x, \qquad\qquad (31a,b,c,d,e)$$

$$\tau'_b = -[(1 + C_a \eta)Tf'']'_{i-1}.$$

Finally, the form in which this equation is solved is

$$[b_5 f'']'_i = b_4 + b_3 f''_i + b_2 f'_i + b_1 f_i, \qquad\qquad (32)$$

where the coefficients are

$$b_1 = - c_4,$$

$$b_2 = c_2 - c_3,$$

$$b_3 = c_1, \qquad\qquad (33a,b,c,d,e)$$

$$b_4 = - \tau'_b + c_1 f''_{i-1} + (c_2 + c_3)f'_{i-1} + c_4 f_{i-1},$$

$$b_5 = - (1 + C_{a_i} \eta)T_i.$$

The corresponding coefficients of equation (32) for the similarity starting flows are

$$b_1 = 0.0,$$

$$b_2 = \underset{\sim}{P}(f' - 2) + (\underset{\sim}{\gamma}/\kappa)Q\,(1 - f'),$$

$$b_3 = (\underset{\sim}{Q} + \underset{\sim}{R})\,[\eta(1 + \underset{\sim}{\gamma}/\kappa) - f] - \underset{\sim}{\gamma}/\kappa \; \underset{\sim}{R}\eta - (v_w/U), \qquad (34a,b,c,d,e)$$

$$b_4 = 0.0,$$

$$b_5 = - (1 + C_a \eta)\underset{\sim}{T}.$$

- 14 -

where the notation is as in equation (28) and $v_w/U = R_{\delta*}(v_w/U)$ in laminar flow and $\underset{\sim}{v}_w/U = v_w/U$ in turbulent flow.

Because equation (32) is nonlinear, the solution is carried out iteratively. The coefficients b_1 to b_5 are evaluated using the result of the previous iteration. The resulting linear equation is then solved for f' and f''. $\delta*$ is adjusted so that $f(\infty) = 1$ to some specified accuracy. The parameters P, Q, R and C_a are recalculated and the effective viscosity function, T, is recalculated. Then the cycle begins again. (To start the calculation at a new x station, the values of f' and f'' from the previous x station and the parameters P, Q, R, and C_a and the effective viscosity function are calculated based on an extrapolated value of $\delta*$).

Solution of Ordinary Differential Equation

Equation (32) is solved with a fourth order Runge-Kutta method. The Runge-Kutta method is a procedure for solving first order differential equations. In order to apply it to equation (32), the equation must be rewritten as a set of first order equations as follows:

Let

$$f^{(1)} = f, \quad f^{(2)} = f', \quad f^{(3)} = b_5 f''.$$

Then

$$\frac{\partial f'}{\partial \eta} = f^{(2)}, \quad \frac{\partial f^{(2)}}{\partial \eta} = \frac{f^{(3)}}{b_5},$$

$$\frac{\partial f^{(3)}}{\partial \eta} = b_4 + b_3 \left(\frac{f^{(3)}}{b_5}\right) + b_2 f^{(2)} + b_1 f^{(1)}.$$

From Hildebrand [14], if the values of $f^{(1)}$ at η_j are known, the values at η_{j+1} ($= \eta_j + \Delta\eta$) are given to fourth order accuracy by the relation

$$f^{(k)}_{j+1} = f^{(k)}_j + 1/6 \left(a_1^{(k)} + 2a_2^{(k)} + 2a_3^{(k)} + a_4^{(k)}\right), \quad (35a)$$

where

$$a_1^{(1)} = \Delta\eta f^{(2)} \quad , \quad a_1^{(2)} = \Delta\eta \left(\frac{f^{(3)}}{b_{5_j}} \right) \quad ,$$

$$a_2^{(1)} = \Delta\eta \left(f^{(2)} + \tfrac{1}{2}a_1^{(2)} \right), \quad a_2^{(2)} = \Delta\eta \left(\frac{f^{(3)} + \tfrac{1}{2}a_1^{(3)}}{b_{5_{j+\frac{1}{2}}}} \right),$$

$$a_3^{(1)} = \Delta\eta \left(f^{(2)} + \tfrac{1}{2}a_2^{(2)} \right), \quad a_3^{(2)} = \Delta\eta \left(\frac{f^{(3)} + \tfrac{1}{2}a_2^{(3)}}{b_{5_{j+\frac{1}{2}}}} \right),$$

$$a_4^{(1)} = \Delta\eta \left(f^{(2)} + a_3^{(2)} \right), \quad a_4^{(2)} = \Delta\eta \left(\frac{f^{(3)} + a_3^{(3)}}{b_{5_{j+1}}} \right),$$

$$a_1^{(3)} = \Delta\eta \left[b_{4_j} + b_{3_j} \left(\frac{f^{(3)}}{b_{5_j}} \right) + b_{2_j} f^{(2)} + b_{1_j} f^{(1)} \right],$$

$$a_2^{(3)} = \Delta\eta \left[b_{4_{j+\frac{1}{2}}} + b_{3_{j+\frac{1}{2}}} \left(\frac{f^{(3)} + \tfrac{1}{2}a_1^{(3)}}{b_{5_{j+\frac{1}{2}}}} \right) + b_{2_{j+\frac{1}{2}}} \left(f^{(2)} + \tfrac{1}{2}a_1^{(2)} \right) \right.$$
$$\left. + b_{1_{j+\frac{1}{2}}} \left(f^{(1)} + \tfrac{1}{2}a_1^{(1)} \right) \right],$$

$$a_3^{(3)} = \Delta\eta \left[b_{4_{j+\frac{1}{2}}} + b_{3_{j+\frac{1}{2}}} \left(\frac{f^{(3)} + \tfrac{1}{2}a_2^{(3)}}{b_{5_{j+\frac{1}{2}}}} \right) + b_{2_{j+\frac{1}{2}}} \left(f^{(2)} + \tfrac{1}{2}a_2^{(2)} \right) \right.$$
$$\left. + b_{1_{j+\frac{1}{2}}} \left(f^{(1)} + \tfrac{1}{2}a_2^{(1)} \right) \right],$$

$$a_4^{(3)} = \Delta\eta \left[b_{4_{j+1}} + b_{3_{j+1}} \left(\frac{f^{(3)} + a_3^{(3)}}{b_{5_{j+1}}} \right) + b_{2_{j+1}} \left(f^{(2)} + a_3^{(2)} \right) \right.$$
$$\left. + b_{1_{j+1}} \left(f^{(1)} + a_3^{(1)} \right) \right], \tag{35b}$$

$$b_{j+\frac{1}{2}} = \tfrac{1}{2} \left(b_j + b_{j+1} \right).$$

The boundary conditions for the solution are split (conditions (13a) and (13b) are applied at the wall and (15) is applied at the outer edge of the layer). Since the Runge-Kutta method requires three boundary conditions at the wall, advantage is taken of the operational linearity of equation (32). Both a homogeneous and a particular solution are calculated which satisfy the following boundary conditions at the wall:

$$f_p''(x,o) = f''(x,o), \quad \text{(From the previous iteration)} \text{**}$$

$$f_p'(x,o) = 1.0, \quad \quad \quad (36a,b,c)$$

$$f_p(x,o) = 0.0$$

for the particular solution and,

$$f_h''(x,o) = 1.0,$$

$$f_h'(x,o) = 0.0,$$

$$f_h(x,o) = 0.0, \quad \quad \quad (37a,b,c)$$

for the homogenous solution. Then a composite numerical solution is constructed according to the relation

$$f' = f_p' + A_f f_h'. \quad \quad \quad (38)$$

A_f is a free parameter which is determined by matching the outer boundary condition. The outer boundary condition is the asymptotic solution, equation (15), which insures the behavior required by equation (13c). From equation (15), at some point η_a, near the edge of the layer,

$$f''(\eta_a) = - A_s \, f'(\eta_a), \quad \quad \quad (39a)$$

where

$$A_s = \frac{(\eta - 1)}{s(x)\,(1+C_a\eta)} \left[1 - \frac{(\eta - 1)C_a}{2(1+C_a\eta)} \right]. \quad \quad \quad (39b)$$

** $f_p''(x,o)$ is reset to $f''(x,o)$ each time so that the particular solution becomes progressively closer to the complete solution. Note that this step is not essential but it does seem to result in some increase in accuracy.

The parameter s(x) is obtained from the approximate solution of equation (16).

$$s_i = \frac{2(\Delta x/\bar{\delta}^*)\, \bar{T}_\infty c_s^2 + s_{i-1}\left[1 - (\bar{Q} + \bar{R})\, c_s (\Delta x/\bar{\delta}^*)\right]}{1 + (\bar{Q} + \bar{R})\, c_s (\Delta x/\bar{\delta}^*)} \qquad (40)$$

or for similarity starting flows, s is given by equation (29).

Inserting equation (39) into (38) and rearranging yields,

$$A_f = - \frac{f_p''(\eta_a) + A_s\, f_p'(\eta_a)}{f_h''(\eta_a) + A_s\, f_h'(\eta_a)} \quad . \qquad (41)$$

In the range $(0 < \eta < \eta_a)$, f' (and f, f'') may be calculated from equation (38). The solution can then be extended out to any value of η using equation (15).

Occasionally, the solutions f_p' and f_h' become so large before η_a is reached that equation (38) no longer gives numerically significant results. For example, on a computer carrying numbers of seven significant figures, if f_p' becomes larger than 10^4, f' given by equation (38) will not in general approach zero at the edge' of the boundary layer to three significant figures. To overcome this problem, a recalculation is performed using the same coefficients (33), but resetting the boundary condition (36a). If recalculation fails to produce a small enough f_p' solution, this indicates that changes in the outer portion of the profile will have no noticeable effect on the f_p' solution, near the wall. Therefore, it is permissable to reset f_p'' at some point, η_s, further out in the layer which will allow a more accurate calculation of f_p'. Recalculation may be performed several times with η_s further and further out as f_p' becomes more accurate.

IV. COMPUTER PROGRAM

Program Notation

Insofar as is possible the variable names in the subroutines are the same as those in the main program. It should be noted, however that some variables which are subscripted in the main program are not subscripted in the subroutines although they are referred to by the same names.

A11..A34   Parameters in the Runge-Kutta method given in Section III, $A\ell m = a_m^{(\ell)}$, $\ell = 1,3$; $m = 1,4$.

ASF   Coefficient in asymptotic solution defined by equation (39b).

AT   Value of f' at which asymptotic outer solution is matched.

B(K,J)   Coefficients in equation (32), $B(K,J) = b_K(\eta_J)$.

BB(K)   Dynamic storage in $FILE.

BK   = 0.016, Clauser constant for outer portion of effective viscosity, [K].

BS   Input for initial pressure gradient described in Section V, [B].

C   Constant in equation (20) for laminar similarity solution, [C].

CA   = $2(\delta^*/r_w) \cos \alpha$, $[C_a]$.

CAF   Constant in matching particular and homogeneous solutions at asymptotic matching point, $[A_f]$.

CA1   = $(2/r_w) \cos \alpha$.

CF   = $\tau_w/(\frac{1}{2}\rho U^2)$, skin friction coefficient.

CHI   = $\kappa y\sqrt{\bar\tau/\rho}/\delta^* U$, coordinate normal to wall used in effective viscosity hypothesis, [X].

CHI3   = $(\kappa y\sqrt{\bar\tau/\rho}/\delta^* U)^3$.

CMT   = $\Pi [CO2/CO1]$.

CO1   = $\theta_I R_I U_I^{2+\bar{\bar{H}}}/\theta_{I-1} R_{I-1} U_{I-1}^{2+\bar{\bar{H}}}$, left-hand side of equation (18).

CO2   = $Exp \left\{ \int_{x_{I-1}}^{x_I} (\bar C_f/2 + v_w/U)d(x/\bar\theta) \right\}$, right-hand side of equation (18).

COAL   = $\cos \alpha_1$, cosine of angle of nose of axisymmetric body.

- 19 -

CW   Longitudinal wall curvature.

C1..C4   Coefficients given by equation (31).

DIVJ   Number of divisions of input profile spacing in floating point
          form.

DT $= \int_{0}^{\infty} (\bar{U}-\bar{u})/\bar{U}(r/r_w)d\bar{y}$, displacement thickness in same units as X, $[\delta^*]$.

DTS   Value of displacement thickness before displacement thickness
          is altered to conform to integral momentum equation.

DTX $= d\delta^*/dx$.

DTXM $= \frac{1}{2}\left[(d\delta^*/dx)_i + (d\delta^*/dx)_{i-1}\right]$, intermediate value of DTX, $[\bar{\delta^*_x}]$.

DX $= x_i - x_{i-1}$, x step size, $[\Delta x]$.

DY $= y_{j+1} - y_j$, y step size.

ET $=$ Value of f' at which calculation of profile stops, $[f'_\infty]$.

FBJE $= f_b(\infty)$.

FHPA   Value of homogeneous part of wolution at asymptotic matching
          points.

FJE $= f(\infty)$.

FP $= (U-u)/U$, velocity defect profile, $[f']$.

FPA   Value of particular part of solution at asymptotic matching
          point.

FPB   Known f' profile at previous x station, $[f'_b]$.

FPPW   Velocity defect gradient at wall, $[f''_w]$.

GAM $= \overline{\sqrt{\tau_w/\rho U^2}}$, ratio of friction velocity to freestream velocity, $[\gamma]$.

HB $= (H_i + H_{i-1})/2$, average shape factor.

I   Index of functions of x for present calculation.

IB $= I-1$, index of functions of x for previous calculation.

ID   Dimension of all functions of x.

IO   Input parameter designating flow inside (IO = -1) or outside (10 = 1) of an axisymmetric body.

IOP   Option number for initialization defined in Section V.

IX   Total number of x station calculations read in, (IX$\leq$ ID).

IXA   Total number of x station calculations actually performed.

IXF   Index of first x station calculation moving downstream.

J   Index of functions of y.

JA   Index of indicated asymptotic matching point.

JAM   = JA - 1.

JAP   = JA + 1.

JD   Dimension of all functions of η.

JDIV   Number of subdivisions of input profile values.

JE   Index of last calculated profile value.

JEM   = JEM - 1.

JEP   = JE + 1.

JK   Index of the point at which the outer effective viscosity reaches a constant value (see Figure 2).

JLP   = JL + 1.

JM   Index of actual asymptotic matching point.

JP   = J + 1.

JF, JL   Indices of first and last points in each subdivision of the input profile.

JS   Index of η point at which iteration within $PROFYL begins.

JSS   JS for next recalculation.

JY   Index of largest η value read in, (JY$\leq$ JD).

JYM   JY - 1.

KMI   Maximum number of complete iterations allowed to calculate a profile.

LABEL   Storage for the label which appears on all output.

LOOP   Index of iterations to calculate profile whose maximum value is KMI.

MT $= \int_{o}^{\infty} \bar{u}(\bar{U}-\bar{u})/\bar{U}^2 (r/r_w)d\bar{y}$, momentum thickness in same units as X, $[\theta]$.

OI   Floating point value of IO.

P $= \delta^* U_x/U$, parameter in equation (11), $[P]$.

PB   Value of P at previous x station.

PM $= \frac{1}{2}(P+PB)$, intermediate value of P.

Q $= (U\delta^*)_x/U$, parameter in equation (11), $[\bar{Q}]$.

QR $= Q + R$.

QRM $= QM + RM$

R $r_{w_x} \delta^*/r_w$, parameter in equation (11).

RB   Value of R at previous X station.

RDT $= U\delta^*/\upsilon$, Reynolds number based on displacement thickness.

RL $= UL/\upsilon$, Reynolds number used in laminar similarity solution.

RM $= (R + RB)/2$, intermediate value of R.

RW   Radius of surface of body in same units as X, $[r_w]$.

RX $= r_{w_x}/r_w$.

S   Parameter in asymptotic outer solution.

SB   Value of S at previous x station.

SC   Parameter in asymptotic solution.

SF $= \delta^*/\theta$.

SIG3 $= (6.9)^3$, empirical constant in the effective viscosity for the wall layer.

SK   = 0.41, von Karman constant in empirical effective viscosity.

SW   = Nikuradse [15] sand grain roughness scale in same units as X, $[s_w]$.

SYG   = 10^{n-3}, where n is the number of significant figures maintained by the computer.

TAU   = $\bar{\tau}/\rho U^2$, nondimensional local shear stress.

TPB   = $[\partial(r\bar{\tau}/r_w\rho\bar{U}^2)/\partial\eta]_{i-1}$, shear stress gradient at previous x station.

TURB   Input parameter which indicates what proportion of effective viscosity is laminar and what part turbulent, [T].

U   Free stream velocity at each X station.

UX   = U_x/U.

U+   = u/u_τ, Law of the Wall velocity.

VE   Nondimensional effective viscosity (12), $[T]$.

VEB   Value of VE at edge of layer for previous X station.

VH, VHP   Dynamic storage principally used to calculate the homogeneous
VHPP   part of the solution in $PROFYL.

VP   = v_w/U, in turbulent flow and $(v_w/U)R_{\delta}*$ in laminar flow.

VR   Value of T in outer part of turbulent layer.

VW   = v_w, transpiration rate, in same units as U.

X   Input values of x at which calculations are to be performed.

XT   Convergence criteria specifying maximum allowable variation between results of consecutive iterations.

Y   = Y/δ^*, independent variable normal to the wall, $[\eta]$.

YY   = $\bar{Y}/\delta^*$, in untransformed coordinates, see equation (6).

YPS   Empirical effective roughness scale.

YY+   = yu_τ/v, independent variable in Law of the Wall region.

Y1   = $\eta_a - 1$, variable in asymptotic outer solution.

Y2   = $\eta - 1$, variable in asymptotic outer solution.

- 23 -

Naming Conventions

( )A   Variable evaluated at asymptotic matching point $[( )_a]$.

( )B   Variable at previous X station, $[( )_b]$.

( )E   Function of η evaluated at free stream.

( )P   Derivative with respect to η, $[( )']$.

( )W   Function of η evaluated at wall, $[( )_w]$.

( )1,2   Function of η evaluated at η_j or η_{j+1}.

\$( )   Denotes subroutine of the program.

      The relative locations of the X and Y indices are illustrated in Figure 3.
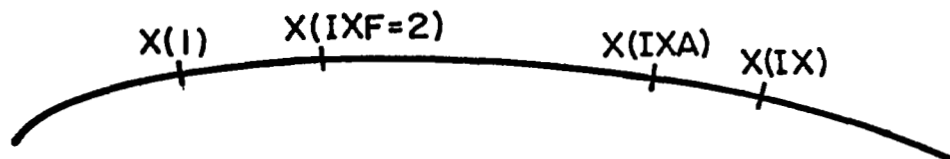
a.) Relative Positions of X Indices for all Cases Except Option 4 of Section V.
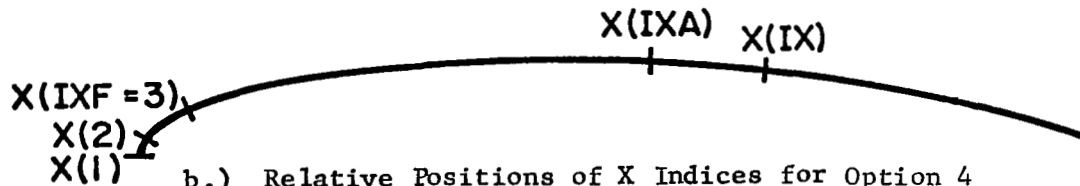
b.) Relative Positions of X Indices for Option 4 of Section V.
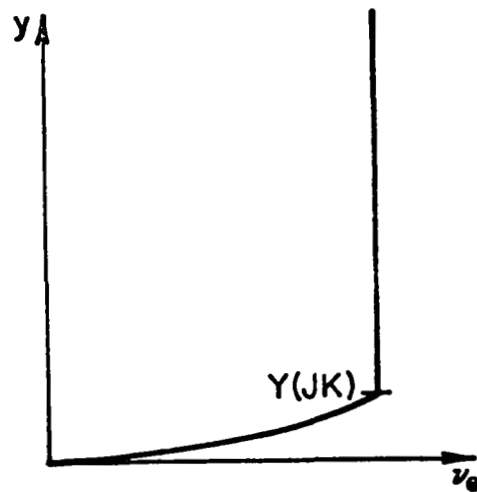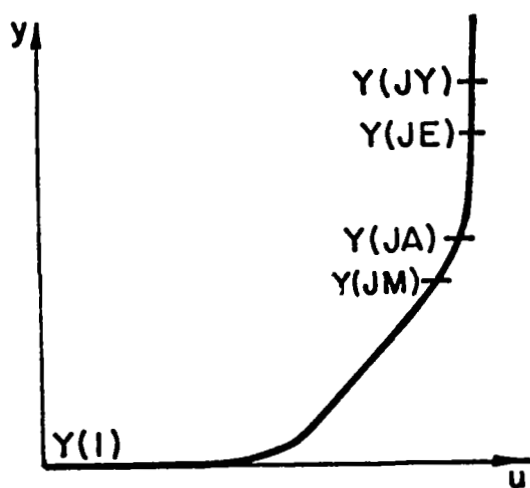
c.) Relative Positions of Y Indices for Turbulent Flow. In Laminar Flow, Since $v_e = v$, JK is Arbitrarily set Equal to JE.

Figure 3. Illustrations of Meaning of Indices Used in X and Y Notation.

Main Program

The program divides naturally into two parts, as is reflected in the flow chart shown in Figure 4. The first part is concerned with the preparation of an appropriate $f'(\eta)$ profile and associated parameters for the initial x station. The second half carries the computation forward to successive x stations. Many of the computational patterns are similar in the two parts and it might seem that the two could be efficiently combined. However, the differences are fundamental enough so that the cause of clarity is best served by keeping them separate.

The first few instructions read in all of the input data required. The appropriate formats for the data and the requirements on the inputs are treated in Section V. Next, the related profiles $f(\eta)$, $f''(\eta)$ and $\tau(\eta)$ are calculated from the input $f'(\eta)$. Then all of the input information and related profiles are printed out for reference. If a similarity solution is to be used to start the boundary layer the input profile is merely a rough guess. In this case the program recalculates the initial profile in the iterative loop which follows. If the input profile is to be used as it is this recalculation is not performed. Finally, in either case, several other initial parameters are calculated and the initial profiles and parameters are printed out in their final form. This is the end of the initialization portion of the program.

The forward motion part of the program consists of a loop which cycles for each x station calculation. The loop begins by moving the known profile into storage for the profile at the x station before the one to be calculated. Then the input boundary conditions are printed out for reference. This is followed by the iterative loop to calculate a new profile at the x station. When this calculation has converged integral parameters for that position are calculated and the integral test for accuracy is printed out. This process continues until profiles have been calculated at all x stations. Finally, for convenience, a summary of the parameters of the flow is printed out.

A listing of the program follows. The numbers in parentheses along the right-hand margin of the listing refer to equation numbers in Sections II and III.

START

READ INITIAL f' (η) PROFILE
AND INITIAL VALUES OF δ^*, R_δ^* AND B

READ IN PARAMETERS FOR EACH X-STATION
U(X), TURB(X), RW(X), V(X), SW(X)

SUBDIVIDE f' (η) BY A FACTOR OF JDIV
AND CALCULATE f(η), f"(η), JA, AND θ

CALCULATE T(η) AND τ(η) FROM THE INPUT PROFILE

PRINT OUT THE INPUT VARIABLES
AND PARAMETERS

LOOP = 1

SET PARAMETERS
DEPENDING ON WHETHER
FLOW IS

LAMINAR   TURBULENT

SET P^*, Q^*, AND R^* SET P, Q, AND R
                                   AND CALCULATE T(η)

IF INPUT
PROFILE IS CORRECT AS IS
SKIP INITIALIZATION
CALCULATION

YES

NO

CALCULATE COEFFICIENTS AND CALL $PROFYL FOR f, f', AND f" SOLUTION

CALCULATE τ(η) AND H

PRINT OUT INTERMEDIATE PARAMETERS
AND VARIABLES

RESET ASYMPTOTIC MATCHING POINT EVERY THIRD TIME

TEST
FOR CONVERGENCE
f(∞) = 1

YES

NO

KMI

WRITE OUT PERTINENT
PARAMETERS

IF INITIALIZATION IS FOR OPTION 4, CALCULATE δ_a^* AND $R_{\delta_a}^*$
AND PRINT EXPLANATION

FOR LAMINAR FLOW CONVERT P^*, Q^*, R^*, AND T^* TO P, Q, R, AND T

CALCULATE δ_x^*, θ, AND C_f

PRINT PROFILES AND
PARAMETERS WITH $FILE

CALCULATE (φf")'

END OF INITIALIZATION

Figure 4a. Flow Chart for the Initialization Section
of the Main Program.

- 27 -

BEGINNING OF FORWARD MOTION
IN X

I = MXF

MOVE f, f', AND f" BACK
TO MOVE FORWARD IN X

PRINT INPUT PARAMETERS FOR NEXT X STATION

LOOP = 1

RECALCULATE δ^*, R_δ^*, P, Q, R, s, AND T(η)

CALCULATE COEFFICIENTS AND CALL $PROFYL FOR f, f', f" SOLUTION

CALCULATE T(η)

WRITE OUT INTERMEDIATE PARAMETERS
AND VARIABLES

TEST FOR
FATAL ERROR,
$F(\infty) < 0.0$ YES

NO

TEST f, f" FOR
CONVERGENCE   YES

NO

KM1

CALCULATE ($\eta f"$)'

COMPUTE H, θ, AND C_f

CALCULATE INTEGRAL OF MOMENTUM EQUATION

PRINT RESULTS
AS AN INDICATION OF ACCURACY,
CO 1 - CO 2

RESET θ, δ^*, AND R_θ^* TO MATCH SKIN FRICTION

RESET ASYMPTOTIC MATCHING POINT, JA

PRINT PROFILES AND
PARAMETERS WITH $FILE

IX

PRINT PROFILES AND
PARAMETERS WHICH
CONTAIN FATAL ERROR

PRINT SUMMARY OF
PRINCIPAL BOUNDARY LAYER PARAMETERS

STOP

Figure 4b. Flow Chart for the Forward Motion Section
of the Main Program.

- 28 -

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C
C
C            THIS COMPUTER PROGRAM PERFORMS A NUMERICAL INTEGRATION
C      OF THE EQUATIONS OF MOTION FOR AN INCOMPRESSIBLE TWO-DIMENSIONAL
C      BOUNDARY LAYER. BOUNDARY LAYER CALCULATIONS MAY EE CARRIED OUT
C      FOR BOTH LAMINAR AND TURBULENT FLOW FOR ARBITRARY REYNOLDS NUMBER
C      AND MAINSTREAM VELOCITY DISTRIBUTION, ON PLANAR OR AXISYMMETRIC
C      BODIES WITH WALL SUCTION OR BLOWING AND WITH A ROUGH OR A SMOOTH
C      WALL.
C
C             GEORGE L. MELLOR AND H. JAMES HERRING
C                      PRINCETON UNIVERSITY
C
       DIMENSION Y(250), YY(250), VE(250), TAU(250), TPB(250), B(5,250)
       DIMENSION F(250), FP(250), FPP(250), FB(250), FPB(250), FPPB(250)
       DIMENSION VH(250), VHP(250), VHPP(250)
       DIMENSION X(60), U(60), TURB(60), RW(60), VW(60), SW(60), CW(60)
       DIMENSION DT(60), AT(60), SF(60), RDT(60), CF(60), LABEL(18)
       REAL MT
       DATA JD, ID, KMI/ 250, 60, 10/
       DATA SK/ 0.41/
       DATA AT, ET, XT/ 0.001, 0.00000001, 0.005/
       WRITE (6,99)
C **** READ INITIAL FP PROFILE AND INITIAL VALUES OF DT, RDT AND B ****
       READ (5,1) (LABEL(K), K=1, 18)
       READ (5,2) JDIV
       READ (5,2) JY
       READ (5,3) (YY(J), J=1, JY)
       READ (5,2) JE
       READ (5,3) (FP(J), J=1, JE)
       READ (5,2) IOP, IO
       READ (5,3) DT(1), RDT(1), BS
C **** READ IN PARAMETERS FOR EACH X STATION ****
       DO 100 I=1, ID
       READ (5,4) X(I), U(I), TURB(I), RW(I), VW(I), SW(I), CW(I)
       IF (X(I).LT.-1000.0) GO TO 101
       IX=I
   100 CONTINUE
   101 CONTINUE
     1 FORMAT (18A4)
     2 FORMAT (16I5)
     3 FORMAT (6F10.5)
     4 FORMAT (7F10.5)
C **** SUBDIVIDE FP BY A FACTOR OF JDIV AND CALCULATE F, FPP, JA AND MT ****
       CALL DIVIDE (JY, JDIV, YY, VH, JD)
       CALL DIVIDE (JE, JDIV, FP, VH, JD)
       I=1
       CMT=1.0
       P=BS
       OI=IO
       R=0.0
       CA=0.0
       SC=1.0
       IF (IO.NE.0) SC=0.5
       GO TO (103,104,104,102,103,104,104), IOP
   102 CONTINUE
       RL=RDT(1)
       C=SQRT(2./(BS+1.)/RL)
```

```
      COAL=SQRT(1.-((RW(2)-RW(1))/(X(2)-X(1)))**2)
      IF (RW(2)/(X(2)-X(1)).LT.1.0E-8) GO TO 105
      R=RL*C*C*(RW(3)-RW(1))/(X(3)-X(1))*(X(2)-X(1))/(RW(3)+RW(1))
      CA=2.*OI*COAL*C*(X(2)-X(1))/RW(2)
      GO TO 105
  103 CONTINUE
      IF (RW(1)/DT(1).LT.0.1) GO TO 105
      R=(RW(2)-RW(1))/(X(2)-X(1))*DT(1)/RW(1)*RDT(1)
      CA=2.*OI*DT(1)/RW(1)*SQRT(1.-((RW(2)-RW(1))/(X(2)-X(1)))**2)
      GO TO 105
  104 CONTINUE
      IF (RW(1)/DT(1).LT.0.1) GO TO 105
      R=(RW(2)-RW(1))/(X(2)-X(1))*DT(1)/RW(1)
      CA=2.*OI*DT(1)/RW(1)*SQRT(1.-R/DT(1)*RW(1)*R/DT(1)*RW(1))
  105 CONTINUE
      DO 110 J=1, JY
      Y(J)=YY(J)*(1.+YY(J)*CA/4.)
  110 CONTINUE
      CALL INTEG (JE, JD, Y, FP, 0.0, F)
      DO 120 J=1, JY
      FPP(J)=(FP(J+1)-FP(J))/(Y(J+1)-Y(J))
      VH(J)=(1.-FP(J))*FP(J)
      VE(J)=1.0
      TAU(J)=0.002
      IF (ABS(FP(J)).GT.AT) JA=J
  120 CONTINUE
      FPP(JY)=0.0
      CALL INTEG (JE, JD, Y, VH, 0.0, VH)
      SF(1)=1.0/VH(JE)
      IF (IOP.NE.4) GO TO 125
      DT(1)=0.0
      RDT(1)=0.0
  125 CONTINUE
      MT(1)=DT(1)/SF(1)
      CF(1)=0.0
      IF (TURB(1).LT.1.0) GO TO 140
C **** CALCULATE VE AND TAU FROM INPUT PROFILE ****
      DO 131 K=1, 5
      CALL VIS (JE, JY, JD, YY, TAU, TURB(1), SW(1), RDT(1), DT(1),
     1    CW(1), JK, VE)
      DO 130 J=1, JY
      TAU(J)=-VE(J)*FPP(J)*SQRT(1.+CA*Y(J))
  130 CONTINUE
  131 CONTINUE
      CF(1)=2.*TAU(1)
  140 CONTINUE
C **** PRINT OUT THE INPUT VARIABLES AND PARAMETERS ****
      WRITE (6,99)
      CALL PILE (LABEL, I, YY, F, FP, FPP, VH, VHP, VHPP, VE, TAU,
     1    X, U, TURB, RW, VW, SW, CW, RDT, DT, MT, SF, CF,
     2    JE, JY, JD, ID, JDIV)
      WRITE (6,51) (LABEL(K), K=1,18)
      WRITE (6,52) JDIV
      WRITE (6,53) ET, AT
      WRITE (6,54) IOP, IO
      WRITE (6,55) DT(1), RL, BS
      WRITE (6,60) X(1), U(1), TURB(1), RW(1), VW(1), SW(1), CW(1)
   51 FORMAT (1H0, 9X, 20HINPUT VARIABLES FOR , 18A4)
```

```
   52 FORMAT (10X, 6HJDIV =I2)
   53 FORMAT (10X, 4HET =, 1PE9.2, 2X, 4HAT =, E9.2)
   54 FORMAT (10X, 5HIOP =, 1X, 4HIO =, I2)
   55 FORMAT (10X, 4HDT =, 1PE9.2, 2X, 5HRDT =, E9.2, 2X, 4HBS =,
     1    1PE9.2)
   60 FORMAT (1H0, 9X, 3HX =, 1PE9.2, 2X,
     1    3HU =, E9.2, 2X, 6HTURB =, 0PF5.3, 2X, 4HRW =, 1PE9.2, 2X,
     2    4HVW =, E9.2, 2X, 4HSW =, E9.2, 2X, 4HCW =, E9.2)
   98 FORMAT (1H0)
   99 FORMAT (1H1)
      WRITE (6,70)
      WRITE (6,71)
   70 FORMAT (1H0, 40X,
     1    48HVALUES OF IMPORTANT VARIABLES FOR EACH ITERATION)
   71 FORMAT (1H0, 1X, 2HJK, 2X, 2HJS, 2X, 2HJM, 2X, 2HJA, 2X, 2HJE,
     1    5X, 3HFJE, 6X, 4HFPPW, 8X, 2HSF, 9X, 1HP, 9X, 1HQ, 9X, 1HS,
     2    7X, 3HCAP, 6X, 3HPPA, 7X, 4HFPPA, 6X, 4HFHPA, 6X, 5HFHPPA/1H0)
C **** BEGINNING OF LOOP TO GENERATE NEW INITIAL FP PROFILE ****
      DO 399 LOOP=1, KMI
C **** SET PARAMETERS DEPENDING ON WHETHER FLOW IS LAMINAR OR TURBULENT ****
      GO TO (303,309,310,300,303,309,310), IOP
  300 CONTINUE
C **** SET P*, Q* AND R* ***
      C=F(JE)*C
      P=C*C*RL*BS                                                  (20a,b)
      Q=C*C*RL*(BS+1.)/2.
      VP=VW(2)/U(2)*C*RL
      R=0.0
      CA=0.0
      IF (RW(2)/(X(2)-X(1)).LT.1.E-8) GO TO 302
      R=RL*C*C*(RW(3)-RW(1))/(X(3)-X(1))*(X(2)-X(1))/(RW(3)+RW(1))  (20c)
      CA=2.*OI*COAL*C*(X(2)-X(1))/RW(2)
      DO 301 J=1, JY
      Y(J)=YY(J)*(1.+YY(J)*CA/4.)
  301 CONTINUE
  302 CONTINUE
      QR=Q+R
      GO TO 305
  303 CONTINUE
      FPP(1)=FPP(1)*F(JE)
      VP=VW(1)/U(1)*RDT(1)
      QR=SF(1)*(-FPP(1)+VP-P)-P                                    (21)
      Q=QR-R
  305 CONTINUE
      S=1./QR*SC                                                   (28)
      GAM=0.0
      GO TO 320
  309 CONTINUE
C **** SET P, Q, R, AND CALCULATE VE ****
      P=-TAU(1)*BS                                                 (26)
  310 CONTINUE
      CF(1)=2.*TAU(1)
      GAM=SQRT(ABS(TAU(1)))
      VP=VW(1)/U(1)
      Q=(SF(1)*(TAU(1)+VP-P)-P-R)/(1.+GAM/SK*(SF(1)-1.))           (27)
      QR=Q+R
      CALL VIS (JE, JY, JD, YY, TAU, TURB(1), SW(1), RDT(1), DT(1),
     1    CW(1), JK, VE)
```

- 31 -

```
      S=VE(JE)/(Q*(1.+GAM/SK)+R)*SC                          (28)
  320 CONTINUE
C **** IF PROFILE IS CORRECT AS IT IS, SKIP INITIALIZATION CALCULATION ****
      IF (IOP.LE.3) GO TO 410
      PPPW=PPP(1)
C **** CALCULATE COEFFICIENTS AND CALL $PROFYL FOR F, PP, FPP SOLUTION ****
      JAP=JA+1
      DO 330 J=1, JAP
      B(1,J)=0.0
      B(2,J)=P*(FP(J)-2.)+GAM/SK*Q*(1.-FP(J))
      B(3,J)=QR*(Y(J)*(1.+GAM/SK)-F(J))-GAM/SK*R*Y(J)-VP      (34a,b,c,d,c)
      B(4,J)=0.0
      B(5,J)=-VE(J)*(1.+CA*Y(J))
  330 CONTINUE
      CALL PROFYL (JE, JA, JY, JD, Y, B, ET, 0.0, 1.0, PPPW, S, CA,
     1   JM, PPA, PPPA, FHPA, FHPPA, CAP, F, PP, FPP, VH, VHP, VHPP)
C **** CALCULATE TAU AND SF ****
      DO 340 J=1, JY
      TAU(J)=-VE(J)*PPP(J)*SQRT(1.+CA*Y(J))
      VH(J)=FP(J)*(1.-FP(J))
  340 CONTINUE
      CALL INTEG (JE, JD, Y, VH, 0.0, VH)
      SF(1)=F(JE)/VH(JE)
C **** PRINT OUT INTERMEDIATE PARAMETERS AND VARIABLES ****
      WRITE (6,72) JK, JM, JA, JE, F(JE), PPP(1), SF(1), P, Q, S,
     1   CAP, PPA, PPPA, FHPA, FHPPA
   72 FORMAT (1X, I3, 5X, 3(I3, 1X), 11(1PE9.2, 1X))
      WRITE (6,98)
C **** RESET ASYMPTOTIC MATCHING POINT EVERY THIRD TIME ****
      IF ((LOOP/3)*3.NE.LOOP) GO TO 395
      DO 394 J=1, JE
      JA=J
      IF (ABS(PP(J)).LT.AT) GO TO 395
  394 CONTINUE
  395 CONTINUE
C **** TEST FOR CONVERGENCE, F(JE)=1.0 ****
      IF (LOOP.GT.2.AND.ABS(F(JE)-1.0).LT.XT) GO TO 400
  399 CONTINUE
C **** END OF LOOP TO GENERATE NEW INITIAL FP PROFILE ****
  400 CONTINUE
      GO TO 415
  410 CONTINUE
C **** PRINT OUT PERTINANT PARAMETERS (IF INITIAL PROFILE IS CORRECT AS IS) ****
      WRITE (6,72) JK, JA, JA, JE, F(JE), PPP(1), SF(1), P, Q, S,
     1   CAP, PPA, PPPA, FHPA, FHPPA
  415 CONTINUE
      IXF=2
      GO TO (430,440,440,420,430,440,440), IOP
  420 CONTINUE
C **** FOR OPTION 4, CALCULATE DT(2), RDT(2) AND PRINT EXPLANATION ****
      IXF=3
      I=2
      SF(2)=SF(1)
      DT(2)=(X(2)-X(1))*SQRT(2.*Q/(BS+1.)/RL)
      RDT(2)=RL*DT(2)/(X(2)-X(1))
      IF (ABS(BS-1.0).LT.0.0001) DT(1)=DT(2)
      MT(1)=DT(1)/SF(1)
      WRITE (6,80) X(2), X(1), DT(1)
```

                              - 32 -

```fortran
   80 FORMAT (1X, 29HTHE FOLLOWING PROFILE AT X = , F10.5, 1X,
      1    42HIS A STARTING LAMINAR SIMILARITY PROFILE.
      2    45HIT IS TAKEN TO BE IDENTICAL TO THE PROFILE AT/
      3    1X, 4HX = , F10.5, 11HWHERE DT = , E9.2)
  430 CONTINUE
C **** FOR LAMINAR FLOW CONVERT P*, Q*, R* AND VE* TO P, Q, R AND VE ****
      P=P/RDT(I)
      Q=Q/RDT(I)
      R=R/RDT(I)
      VP=VP/RDT(I)
      DO 435 J=1, JE
      TAU(J)=-FPP(J)/RDT(I)*SQRT(1.+CA*Y(J))
      VE(J)=VE(J)/RDT(I)
  435 CONTINUE
  440 CONTINUE
C **** CALCULATE DTX, MT AND CF ****
      DTXM=Q-P
      MT(I)=DT(I)/SF(I)
      CF(I)=2.*TAU(1)
C **** PRINT PROFILES AND PARAMETERS WITH $FILE ****
      WRITE (6,99)
      CALL FILE (LABEL, I, YY, F, FP, FPP, VH, VHP, VHPP, VE, TAU,
      1    X, U, TURB, RW, VW, SW, CW, RDT, DT, MT, SF, CF,
      2    JE, JY, JD, ID, JDIV)
      IF (IX-I.LE.0) STOP
C **** CALCULATE TPB ****
      DO 480 J=1, JY
      TPB(J)=((Q+R)*Y(J)*(1.+GAM/SK)-F(J)-GAM/SK*R*Y(J)-VP)*FPP(J)
      1    +(P*(FP(J)-2.)+GAM/SK*Q*(1.-FP(J)))*FP(J)
  480 CONTINUE
C
C
C
C **** END OF INITIALIZATION, BEGINNING OF FORWARD MOTION IN X ****
C
C
      DO 899 I=IXF, IX
      IXA=I
C **** MOVE F, FP, FPP, BACK TO MOVE FORWARD IN X ****
      DO 510 J=1, JY
      FB(J)=F(J)
      FPB(J)=FP(J)
      FPPB(J)=FPP(J)
  510 CONTINUE
      VEB=VE(JE)
      PB=P
      RB=R
      SB=S
      IB=I-1
      DX=X(I)-X(IB)
      DT(I)=DT(IB)+DTXM*DX
      IF (I.GE.IX) GO TO 520
      UX=U(I-1)/U(I)*(X(I)-X(I+1))/(X(I-1)-X(I))/(X(I-1)-X(I+1))
      1    +(2.*X(I)-X(I-1)-X(I+1))/(X(I)-X(I-1))/(X(I)-X(I+1))
      2    +U(I+1)/U(I)*(X(I)-X(I-1))/(X(I+1)-X(I-1))/(X(I+1)-X(I))
      RX=0.0
      CA1=0.0
      IF (RW(I)/DT(I).LT.0.1) GO TO 520
      RX=RW(I-1)/RW(I)*(X(I)-X(I+1))/(X(I-1)-X(I))/(X(I-1)-X(I+1))
```

(32) & (34)

```
      1     +(2.*X(I)-X(I-1)-X(I+1))/(X(I)-X(I-1))/(X(I)-X(I+1))
      2     +RW(I+1)/RW(I)*(X(I)-X(I-1))/(X(I+1)-X(I-1))/(X(I+1)-X(I))
            CA1=2.*OI*SQRT(1.-RX*RW(I)*RX*RW(I))/RW(I)
  520 CONTINUE
C **** PRINT INPUT PARAMETERS FOR NEXT X STATION ****
      WRITE (6,51) (LABEL(K), K=1,18)
      WRITE (6,60) X(I), U(I), TURB(I), RW(I), VW(I), SW(I), CW(I)
      WRITE (6,70)
      WRITE (6,76)
   76 FORMAT (1H0, 1X, 2HJK, 2X, 2HJS, 2X, 2HJM, 2X, 2HJA, 2X, 2HJE,
      1     5X, 3HFJE, 6X, 4HFPPW, 8X, 2HDT, 8X, 2HPM, 8X, 2HQM, 9X, 1HS,
      2     7X, 3HCAF, 6X, 3HFPA, 7X, 4HFPPA, 6X, 4HFHPA, 6X, 5HFHPPA/ 1H0)
C **** BEGIN ITERATIVE LOOP TO CALCULATE PROFILE ****
      DO 799 LOOP=1, KMI
C **** RECALCULATE DT, RDT, P, Q, R, S AND VE ****
      DT(I)=DT(I)*F(JE)
      RDT(I)=RDT(IB)*U(I)/U(IB)*DT(I)/DT(IB)
      DTXM=(DT(I)-DT(IB))/DX
      P=DT(I)*UX
      PM=(P+PB)/2.
      QM=DTXM+PM
      R=RX*DT(I)
      RM=(R+RB)/2.
      QRM=QM+RM
      CA=CA1*DT(I)
      IF (RW(I)/DT(I).LT.0.1) GO TO 602
      DO 601 J=1, JY
      YY(J)=2.*Y(J)/(1.+SQRT(1.+CA*Y(J)))
  601 CONTINUE
  602 CONTINUE
      CALL VIS (JE, JY, JD, YY, TAU, TURB(I), SW(I), RDT(I), DT(I),
      1     CW(I), JK, VE)
      S=(SB*(1.-2.*DX/(DT(I)+DT(IB))*QRM*SC)
      1     +2.*DX/(DT(I)+DT(IB))*(VE(JE)+VEB)*SC*SC)                    (40)
      2     /(1.+2.*DX/(DT(I)+DT(IB))*QRM*SC)
      FPPW=FPP(1)
C **** CALCULATE COEFFICIENTS AND CALL $PROPYL FOR F, FP, FPP SOLUTION ****
      DO 610 J=1, JE
      C1=QRM*(Y(J)-(FB(J)+FB(J))/2.)-(VW(I)+VW(IB))/(U(I)+U(IB))
      C2=PM*((FP(J)+FPB(J))/2.-2.)                                      (31a,b,c,d,e)
      C3=(DT(I)+DT(IB))/DX*(1.-(FP(J)+FPB(J))/2.)
      C4=(DT(I)+DT(IB))/DX*(FPP(J)+FPPB(J))/2.
      B(1,J)=-C4
      B(2,J)=C2-C3
      B(3,J)=C1                                                         (33a,b,c)
      B(4,J)=-TPB(J)+C1*FPPB(J)+(C2+C3)*FPB(J)+C4*FB(J)
      B(5,J)=-VE(J)*(1.+CA*Y(J))                                        (33d,e)
  610 CONTINUE
      CALL PROFYL (JE, JA, JY, JD, Y, B, ET, 0.0, 1.0, FPPW, S, CA,
      1     JM, FPA, FPPA, FHPA, FHPPA, CAF, F, FP, FPP, VH, VHP, VHPP)
C **** CALCULATE TAU ****
      DO 620 J=1, JE
      TAU(J)=-VE(J)*FPP(J)*SQRT(1.+CA*Y(J))
  620 CONTINUE
C **** PRINT OUT INTERMEDIATE PARAMETERS AND VARIABLES ****
      WRITE (6,77) JK, JM, JA, JE, F(JE), FPP(1), DT(I), PM, QM, S,
      1     CAF, FPA, FPPA, FHPA, FHPPA
   77 FORMAT (1X, I3, 5X, 3(I3, 1X), 11(1PE9.2, 1X))
```

```
      WRITE (6,98)
C **** TEST FOR FATAL ERROR, FJE LT 0.0 ****
      IF (F(JE).LT.0.0.OR.F(JE).GT.10.0) GO TO 901
C **** TEST F, FPP FOR CONVERGENCE ****
      IF (LOOP.EQ.1) GO TO 798
      IF (ABS((FPP(1)-FPPW)/FPPW).GT.XT) GO TO 798
      IF (ABS(F(JE)-1.0).LT.XT) GO TO 800
  798 CONTINUE
  799 CONTINUE
C **** END OF ITERATIVE LOOP TO CALCULATE PROFILE ****
  800 CONTINUE
C **** CALCULATE TPB ****
      DO 810 J=1, JE
      TPB(J)=B(4,J)+B(3,J)*FPP(J)+B(2,J)*FP(J)+B(1,J)*F(J)           (32)
  810 CONTINUE
      JEP=JE+1
      DO 811 J=JEP, JY
      TPB(J)=0.0
  811 CONTINUE
C **** COMPUTE SF, MT AND CF ****
      DO 840 J=1, JE
      VH(J)=FP(J)*(1.-FP(J))
  840 CONTINUE
      CALL INTEG (JE, JD, Y, VH, 0.0, VH)
      SF(I)=1./VH(JE)
      MT(I)=DT(I)/SF(I)
      CF(I)=2.*TAU(1)
C **** CALCULATE INTEGRAL OF EQUATION ****
      HB=(SF(I)+SF(IB))/2.
      CO1=(U(I)/U(IB))**(2.+HB)*MT(I)/MT(IB)
      IF (RW(IB).GT.DT(IB)/10.0) CO1=CO1*RW(I)/RW(IB)
      CO2=EXP(DX*((CF(IB)+CF(I))/2.+2.*(VW(IB)+VW(I))/(U(IB)+U(I)))
     1    /(MT(IB)+MT(I)))
      CMT=CMT*CO2/CO1                                                (18)
C **** PRINT RESULT AS AN INDICATION OF ACCURACY, CO1 = CO2 ****
      WRITE (6,85) CO1, CO2, CMT
   85 FORMAT (1H0, 30X, 29HINTEGRAL OF MOMENTUM EQUATION , 3X,
     1    29HCUMULATIVE MOMENTUM INTEGRAL // 37X, F7.4, 4H  = , F7.4,
     2    20X, F8.5)
C **** RESET MT, DT AND RDT TO MATCH THE SKIN FRICTION ****
      DTS=DT(I)
      MT(I)=MT(I)*CO2/CO1
      DT(I)=SF(I)*MT(I)
      RDT(I)=RDT(I)/DTS*DT(I)
      DTXM=(DT(I)-DT(IB))/DX
C ****RESET ASYMPTOTIC MATCHING POINT, JA ****
      DO 890 J=1, JE
      IF (ABS(FP(J)).LT.AT) GO TO 891
      JA=J
  890 CONTINUE
  891 CONTINUE
C **** PRINT PROFILES AND PARAMETERS WITH $FILE ****
      WRITE (6,99)
      CALL FILE (LABEL, I, YY, F, FP, FPP, VH, VHP, VHPP, VE, TAU,
     1    X, U, TURB, RW, VW, SW, CW, RDT, DT, MT, SF, CF,
     2    JE, JY, JD, ID, JDIV)
  899 CONTINUE
C **** END OF FORWARD MOTION IN X ****
```

```
          GO TO 905
      901 CONTINUE
C **** PRINT PROFILES AND PARAMETERS WHICH CONTAIN FATAL ERROR, PJE LT 0.0 ****
          WRITE (6,99)
          CALL FILE (LABEL, I, YY, F, FP, FPP, VH, VHP, VHPP, VE, TAU,
         1    X, U, TURB, RW, VW, SW, CW, RDT, DT, MT, SF, CF,
         2    JE, JY, JD, ID, JDIV)
      905 CONTINUE
C **** PRINT OUT SUMMARY OF PRINCIPAL BOUNDARY LAYER PARAMETERS ****
          WRITE (6,90) (LABEL(K), K=1,18)
          WRITE (6,91)
          DO 910 I=1, IXA
          WRITE (6,92) X(I), DT(I), MT(I), SF(I), CF(I), RDT(I), U(I),
         1    TURB(I), RW(I), VW(I), SW(I), CW(I)
      910 CONTINUE
          WRITE (6,99)
       90 FORMAT (//45X, 39HPRINCIPAL BOUNDARY LAYER PARAMETERS FOR/
         1    26X, 18A4)
       91 FORMAT (// 9X, 1HX, 9X, 2HDT, 8X, 2HMT, 6X, 2HSF, 7X, 2HCF, 8X,
         1    3HRDT, 8X, 1HU, 6X, 4HTURB, 6X, 2HRW, 7X, 2HVW, 8X, 2HSW,
         2    8X, 2HCW)
       92 FORMAT (/3X, 3(1X, F9.4), 1X, F7.4, 1X, F9.6, 1X, F9.0, 1X,
         1    F9.4, 1X, F6.3, 1X, F9.4, 1X, F9.6, 1X, F9.6, 1X, F9.6)
          STOP
          END
```

Description of the Subroutines

Subroutine VIS calculates the effective viscosity, T, in terms of the local flow variables. The entire effective viscosity hypothesis for turbulent flow is contained in this subroutine so that changes can be made without affecting the main program.

In laminar flow T is merely $1./R_\delta^*$. In turbulent flow T is given by

$$T = \varphi(R_\delta^* X)/R_\delta^* + \Phi(X) - X \qquad (42)$$

where $X = \kappa y \sqrt{\tau/\rho}/U\delta^*$. The form of the functions φ and Φ is shown in Figure 2. A more complete discussion of this hypothesis is given in Reference [1].

A method of simulating the effects of wall roughness in turbulent flow is provided and is completely contained in $VIS. This is accomplished by adding a quantity y_s^+ to X in Figure 2a so that $\varphi(X)$ is modified to $\varphi(X + y_s^+)$. y_s^+ is then functionally related to $s_w u_\tau/v$ so that predicted profiles conform to Figures 20.21 in Reference [11] where $k_s = s_w$ is Nikuradse's roughness height. For small $s_w u_\tau/v$ one obtains hydraulically smooth predictions whereas for large $s_w u_\tau/v$, hydraulically rough predictions are obtained. In any event, the direct effect of roughness is to displace downward the logarithmic portion of the law of the wall and to increase the skin friction coefficient.

Finally, a very minimal mechanism for causing transition is provided, again entirely within this subroutine. This mechanism allows the user to specify the relative proportions of laminar and turbulent kinematic viscosity which will prevail according to the relation

$$T = T\ T_{turbulent} + (1-T)\ T_{laminar}, \quad (0 < T < 1). \qquad (43)$$

In completely laminar flow T = 0 and in fully turbulent flow T = 1.

Classification of Arguments
    Inputs: JE, JY, JD, YY, TAU, TURB, SW, RDT, DT, CW.
    Outputs: JK, VE.

```
      SUBROUTINE VIS (JE, JY, JD, YY, TAU, TURB, SW, RDT, DT, CW,
     1   JK, VE)
      DIMENSION YY(JD), TAU(JD), VE(JD)
      DATA SIG3, SK, BK/328.51, 0.41, 0.016/
      JK=JE
      GAM=SQRT(ABS(TAU(1)))
      IF (TURB.LT.1.0E-10) GO TO 300
      VR=BK
      YPS=SW/DT*RDT*GAM/30.0*(1.0+3.0*EXP(-SW/DT*RDT*GAM/150.0))
      DO 200 J=1, JY
      JK=J
      CHI=SK*(RDT*YY(J)*SQRT(ABS(TAU(J)))+YPS)
      CHI3=CHI*CHI*CHI
      VE(J)=(1.+CHI3*CHI/(CHI3+SIG3))/RDT
      IF (VE(J).GT.VR) GO TO 210
200   CONTINUE
      GO TO 300
210   CONTINUE
      DO 220 J=JK, JY
      VE(J)=VR
220   CONTINUE
300   CONTINUE
      DO 310 J=1, JY
      VE(J)=TURB*VE(J)+(1.-TURB)/RDT
310   CONTINUE
      RETURN
      END
```

Subroutine PROFYL controls the numerical solution of the ordinary differential equation (32). Actually, as described in Section III, two solutions are obtained corresponding to the homogeneous and the particular forms of the equation. These are then combined to form the complete solution so that the boundary conditions (13) are satisfied. The asymptotic outer solution is also calculated so that the output is the complete f' profile.

Classification of Arguments

    Inputs: JE, JA, JY, JD, Y, B, ET, FW, FPW, FPPW, S, CA.
    Outputs: JM, FPA, FPPA, FHPA, FHPPA, CAF, F, FP, FPP, VH, VHP, VHPP.

```
      SUBROUTINE PROFYL (JE, JA, JY, JD, Y, B, ET, FW, FPW, FPPW, S, CA,
     1   JM, FPA, FPPA, FHPA, FHPPA, CAF, F, FP, FPP, VH, VHP, VHPP)
      DIMENSION Y(JD), B(5,JD)
      DIMENSION F(JD), FP(JD), FPP(JD), VH(JD), VHP(JD), VHPP(JD)
      SYG=1.0E3
      FPPWS=FPPW
      F(1)=FW
      FP(1)=FPW                                                      (36a,b,c)
      FPP(1)=FPPW
      VH(1)=0.0
      VHP(1)=0.0                                                     (37a,b,c)
      VHPP(1)=1.0
      JAM=JA-1
      DO 200 J=1, JAM
      JP=J+1
      DY=Y(JP)-Y(J)
      CALL RUNGE (B(1,J), B(2,J), B(3,J), B(4,J), B(5,J),
     1   B(1,JP), B(2,JP), B(3,JP), B(4,JP), B(5,JP),
     2   DY, F(J), FP(J), FPP(J), F(JP), FP(JP), FPP(JP))
      CALL RUNGE (B(1,J), B(2,J), B(3,J), 0.0, B(5,J),
     1   B(1,JP), B(2,JP), B(3,JP), 0.0, B(5,JP),
     2   DY, VH(J), VHP(J), VHPP(J), VH(JP), VHP(JP), VHPP(JP))
      IF (ABS(FP(JP)).LT.SYG) JM=JP
  200 CONTINUE
      JSS=1
      IF (JM.GE.JA) GO TO 230
      DO 225 K=1, 5
      JS=JSS
      ASF=(Y(JM)-1.)/S/(1.+CA*Y(JM))
     1   *(1.-(Y(JM)-1.)*CA/2./(1.+CA*Y(JM)))                        (39b)
      CAF=-(FPP(JM)+ASF*FP(JM))/(VHPP(JM)+ASF*VHP(JM))               (41)
      DO 215 J=1, JS
      F(J)=F(J)+CAF*VH(J)
      FP(J)=FP(J)+CAF*VHP(J)                                        (38)
      FPP(J)=FPP(J)+CAF*VHPP(J)
  215 CONTINUE
      WRITE (6,10) JS, JM, JA, JE, FPPWS, CAF, FP(JM), FPP(JM),
     1   VHP(JM), VHPP(JM)
   10 FORMAT (5X, 4(I3, 1X), 10X, 1PE9.2, 41X, 5(1PE9.2, 1X))
      FPPWS=FPP(1)
      DO 220 J=JS, JAM
      JP=J+1
      DY=Y(JP)-Y(J)
      CALL RUNGE (B(1,J), B(2,J), B(3,J), B(4,J), B(5,J),
     1   B(1,JP), B(2,JP), B(3,JP), B(4,JP), B(5,JP),
     2   DY, F(J), FP(J), FPP(J), F(JP), FP(JP), FPP(JP))
      IF (ABS(FP(JP)).LT.ABS(FP(J))) JSS=JP
      IF (ABS(FP(JP)).GT.SYG) GO TO 225
      JM=JP
  220 CONTINUE
      GO TO 230
  225 CONTINUE
  230 CONTINUE
      ASF=(Y(JM)-1.)/S/(1.+CA*Y(JM))
     1   *(1.-(Y(JM)-1.)*CA/2./(1.+CA*Y(JM)))                        (39b)
      CAF=-(FPP(JM)+ASF*FP(JM))/(VHPP(JM)+ASF*VHP(JM))               (41)
      IF (ABS(FP(JM)+CAF*VHP(JM)).LT.ABS(FP(JM-1)+CAF*VHP(JM-1)))
     1   GO TO 240
```

```
      JM=JM-1
      GO TO 230
  240 CONTINUE
      FPA=FP(JM)
      FPPA=PPP(JM)
      FHPA=VHP(JM)
      FHPPA=VHPP(JM)
      DO 250 J=1, JM
      F(J)=F(J)+CAF*VH(J)
      FP(J)=FP(J)+CAF*VHP(J)                                    (38)
      FPP(J)=FPP(J)+CAF*VHPP(J)
  250 CONTINUE
      Y1=(Y(JM)-1.)**2/(1.+CA*Y(JM))
      DO 260 J=JM, JY
      JE=J
      Y2=Y(J)-1.
      FP(J)=FP(JM)*EXP((Y1-Y2*Y2/(1.+CA*Y(J)))/2./S)           (15)
      F(J)=F(J-1)+(FP(J)+FP(J-1))*(Y(J)-Y(J-1))/2.
      FPP(J)=-Y2/S/(1.+CA*Y(J))*(1.-Y2*CA/2./(1.+CA*Y(J)))*FP(J)  (39a)
      IF (ABS(FP(J)).LT.ET) GO TO 270
  260 CONTINUE
  270 CONTINUE
      DO 280 J=JE, JY
      F(J)=F(JE)
      FP(J)=0.0
      FPP(J)=0.0
  280 CONTINUE
      RETURN
      END
```

Subroutine RUNGE is a subroutine called by $PROFYL which contains the actual Runge-Kutta equations (equations 35a,b, in Section III), which are used to calculate both homogeneous and particular solutions for each step in η.

Classification of Arguments
Inputs: B11, B12, B13, B14, B15, B21, B22, B23, B24, B25, BY,
          F1, FP1, FPP1.

Outputs: F2, FP2, FPP2.

```
      SUBROUTINE RUNGE (B11, B21, B31, B41, B51, B12, B22, B32, B42,
     1   B52, DY, F1, FP1, FPP1, F2, FP2, FPP2)
C
C          THIS SUBROUTINE PERFORMS ONE STEP OF A FOURTH ORDER RUNGE-
C     KUTTA INTEGRATION FOR THE EQUATION
C                    (B5*FPP) P=B3*FPP+B2*FP+B1*F+B4
C     B11..B51 AND B12..B52 ARE THE REQUIRED COEFFICIENTS AT Y(J) AND
C     Y(J)+DY RESPECTIVELY, WHEREAS F1, FP1, FPP1 AND F2, FP2, FPP2 ARE
C     THE DEPENDENT VARIABLES AND THEIR DERIVITIVES.
      TAU1=B51*FPP1
      A11=DY*FP1
      A21=DY*TAU1/B51
      A31=DY*(B31*TAU1/B51+B21*FP1+B11*F1+B41)
      A12=DY*(FP1+A21/2.)
      A22=DY*(TAU1+A31/2.)/((B51+B52)/2.)
      A32=((B31+B32)/2.)*A22+((B21+B22)/2.)*A12               (35b)
     1   +DY*((B11+B12)/2.*(F1+A11/2.)+(B41+B42)/2.)
      A13=DY*(FP1+A22/2.)
      A23=DY*(TAU1+A32/2.)/((B51+B52)/2.)
      A33=((B31+B32)/2.)*A23+((B21+B22)/2.)*A13
     1   +DY*((B11+B12)/2.*(F1+A12/2.)+(B41+B42)/2.)
      A14=DY*(FP1+A23)
      A24=DY*(TAU1+A33)/B52
      A34=B32*A24+B22*A14+DY*(B12*(F1+A13)+B42)
      F2=F1+(A11+2.*A12+2.*A13+A14)/6.
      FP2=FP1+(A21+2.*A22+2.*A23+A24)/6.                      (35a)
      TAU2=TAU1+(A31+2.*A32+2.*A33+A34)/6.
      FPP2=TAU2/B52
      RETURN
      END
```

Subroutine <u>INTEG</u> performs a simple trapazoidal quadrature.

Classification of Arguments
     Inputs: JE, JD, Y, FP, FIRST.
     Outputs: F.

```
      SUBROUTINE INTEG (JE, JD, Y, FP, FIRST, F)
      DIMENSION Y(JD), FP(JD), F(JD)
      JEM=JE-1
      FP2=FP(1)
      F1=FIRST
      F(1)=F1
      DO 110 J=1, JEM
      FP1=FP2
      FP2=FP(J+1)
      F1=F1+(Y(J+1)-Y(J))*(FP2+FP1)/2.
      F(J+1)=F1
  110 CONTINUE
      IF (JE.GE.JD) RETURN
      DO 120 J=JE, JD
      F(J)=F(JE)
  120 CONTINUE
      RETURN
      END
```

Subroutine <u>DIV</u> subdivides the interval between the values which define a function using a linear interpolation.

Classification of Arguments
        Inputs: JY, JDIV, Y, VH, JD.
        Outputs: JY, Y.

```
      SUBROUTINE DIVIDE (JY, JDIV, Y, VH, JD)
      DIMENSION Y(JD), VH(JD)
      DO 110 J=1, JY
      VH(J)=Y(J)
110   CONTINUE
      DIVJ=JDIV
      JYM=JY-1
      JY=JYM*JDIV
      DO 130 J=1, JYM
      DY=(VH(J+1)-VH(J)))/DIVJ
      JP=(J-1)*JDIV+2
      JL=J*JDIV
      Y(JP-1)=VH(J)
      DO 120 J1=JP, JL
      Y(J1)=Y(J1-1)+DY
120   CONTINUE
130   CONTINUE
      IF (JL+1.LE.JD) Y(JY+1)=VH(JYM+1)
      IF (JL+1.GE.JD) RETURN
      JLP=JL+2
      DO 140 J=JLP, JD
      Y(J)=0.0
140   CONTINUE
      RETURN
      END
```

Subroutine FILE prints out the profiles and parameters at each x station.

Classification of Arguments
    Inputs: LABEL, I, YY, F, FP, FPP, VH, VHP, VHPP, VE, TAU, X, U,
                TURB, RW, VW, SW, CW, RDT, DT, MT, SF, CF, JE, JY, JD, ID, JDIV.
    Outputs: NONE.

```
      SUBROUTINE FILE (LABEL, I, YY, F, FP, FPP, VH, VHP, VHPP, VE, TAU,
     1   X, U, TURB, RW, VW, SW, CW, RDT, DT, MT, SF, CF,
     2   JE, JY, JD, ID, JDIV)
      REAL MT
      DIMENSION YY(JD), VE(JD), TAU(JD), BB(13), LABEL(18)
      DIMENSION F(JD), FP(JD), FPP(JD), VH(JD), VHP(JD), VHPP(JD)
      DIMENSION X(ID), U(ID), RDT(ID), TURB(ID), RW(ID), VW(ID), SW(ID),
     1   CW(ID)
      DIMENSION DT(ID), MT(ID), SF(ID), CF(ID)
      WRITE (6,10) (LABEL(K), K=1,18)
      WRITE (6,11) X(I), U(I), RDT(I)
      WRITE (6,12) TURB(I), RW(I), VW(I), SW(I), CW(I)
      WRITE (6,13) DT(I), MT(I), CF(I), SF(I)
      WRITE (6,20)
      GAM=SQRT(ABS(CF(I)/2.))
      IF (GAM.LT.1.0E-6) GAM=1.0
      DO 110 J=1, JE, JDIV
      BB(1)=YY(J)
      BB(2)=1.-FP(J)
      BB(3)=YY(J)*RDT(I)*GAM
      BB(4)=(1.-FP(J))/GAM
      BB(5)=VE(J)
      BB(6)=TAU(J)
      BB(7)=F(J)
      BB(8)=FP(J)
      BB(9)=FPP(J)
      BB(10)=VH(J)
      BB(11)=VHP(J)
      BB(12)=VHPP(J)
      BB(13)=YY(J)*SF(I)
      WRITE (6,21) J, (BB(K), K=1,13)
  110 CONTINUE
      WRITE (6,9)
      RETURN
    9 FORMAT (1H1)
   10 FORMAT (1H0, 46X, 27HBOUNDARY LAYER PROFILES FOR //26X, 18A4//)
   11 FORMAT (1H0, 37X, 3HX =, F8.3, 3X, 3HU =, F8.3,
     1   3X, 5HRDT =, 1PE9.2)
   12 FORMAT (1H0, 25X, 6HTURB =, F5.3, 4X, 4HRW =, 1PE9.2, 4X,
     1   4HVW =, E9.2, 4X, 4HSW =, E9.2, 4X, 4HCW =, E9.2)
   13 FORMAT (1H0, 31X, 4HDT =, 1PE9.2, 3X,
     1   4HMT =, E9.2, 3X, 4HCF =, E9.2, 3X, 4HSF =, E9.2)
   20 FORMAT (1H0, 4X, 1HJ, 6X, 2HYY, 7X, 3HU/U, 6X, 3HYY+, 6X, 2HU+,
     1   7X, 2HVE, 6X, 3HTAU, 7X, 1HF, 7X, 2HFP, 8X, 3HFPP,
     2   7X, 3H  , 7X, 3H  , 7X, 5X, 5HYY*SF)
   21 FORMAT (4X, I3, 1X, 7(1PE9.2), 4(E9.2, 1X), 2E9.2)
      END
```

- 44 -

V. PROGRAM OPERATION

Input

Just as the program itself falls into two parts, the input to the program is best considered in two parts. The first part is the choice of the method of initialization best suited to a specific problem. Here the decision is made on the basis of which method best represents reality. The second part of the input concerns the downstream calculation. There is considerably more latitude in the specification of how this is to be carried out.

The input to the initialization section begins with a title, written in the first 72 columns of the first card. This title is printed as a label on all of the output produced by the program

```
| LABEL DESCRIBING CALCULATION                                                    |
|.................................................................................|
```

The first group of numerical inputs apply to the distribution of η points and the input $f'(\eta)$ profile. To save the effort on preparing a large number of input values, provision has been made in the program to subdivide the input values of η and $f'(\eta)$. The number of subdivisions between each input value is specified by JDIV. The total number of η values which is to follow is JY. JY and JDIV must be such that the product of JDIV and JY is less than JD. Then the η values are listed 6 to a card. These are followed by JE, defining the total number of profile values f' and finally the f' values listed 6 to a card.

```
| JDIV                                                                            |
|.................................................................................|
```

```
| JY                                                                              |
|.................................................................................|
```

```
|  Y(1)  |  Y(2)  |  Y(3)  |  Y(4)  |  Y(5)  |  Y(6)  |                            |
|.................................................................................|
```

```
|  Y(7)  |  Y(8)  |    •    |    •    |    •    |    •    |                        |
|.................................................................................|
```

```
| JE                                                                              |
|.................................................................................|
```

```
| FP(1)  | FP(2)  | FP(3)  | FP(4)  | FP(5)  | FP(6)  |                           |
|.................................................................................|
```

```
| FP(7)  | FP(8)  |    •    |    •    |    •    |    •    |                        |
|.................................................................................|
```

Since the η step size is fixed, the input η spacing, when subdivided by JDIV, should be adequate to define a profile. In a wholly laminar calculation the η spacing need not vary appreciably across the layer but in a calculation with a turbulent portion, smaller spacing should be prescribed close to the wall than is specified further out, in order to resolve details both viscous and logarithmic in portions of the law of the wall region. The outer edge of the boundary layer in η coordinates will not move in or out appreciably as the calculations proceed downstream because η has been normalized with δ^*. However, fineness of η spacing near the wall will be conditioned by the largest Reynolds number encountered in a calculation. A few sample calculations should provide the necessary experience. The samples of η distributions presented in these examples should cover most cases, however. Finally, for very small x-step size, smaller η spacing will be required throughout the layer.

In the specification of the initial f' profile, the requirements differ greatly depending on whether the input profile is to be used without change or is to be recalculated. If the input profile is to be used as is, some confidence in its compatibility with the initial pressure gradient is assumed. For a turbulent flow, the profile must be defined in the law of the wall region. If the profile is known from experiments, for instance, and as is frequently the case, only the outer part is known with confidence, then it is best to supply the required points close to the wall from the widely accepted empirical "law of the wall". On the other hand, if the initial profile is to be recalculated to obtain a similarity solution, it may be a rough guess; the calculation of the similarity solution converges strongly to a profile independent of the input profile.

The next few cards are concerned with the method of initialization itself. The variety of situations for which the program has been implemented is given in the following outline, along with the appropriate sets of input cards for each. Their theoretical basis has been discussed in Section II.

The first parameter on the first card of each group, IOP, is the option number as assigned in the following outline. The second parameter, IO, designates whether an axisymmetric flow is on the inside (IO = -1) or the outside (IO = 1) of the surface. For plane flow IO need not be specified.

THE INPUT PROFILE IS TO BE USED AS IS.

Laminar Flow
Option 1, initial δ^*, R_{δ^*}, and $U_x \delta^{*2}/\nu$ are known.

1	IO	

DT(1)	RDT(1)	P	

X(1)	U(1)	0.0	RW(1)	VW(1)	0.0	CW(1)

- 46 -

Turbulent Flow
    Option 2, initial δ^*, R_{δ^*}, and β are known.

2	ID	

DT(1)	RDT(1)	B	

X(1)	U(1)	1.0	RW(1)	VW(1)	SW(1)	CW(1)

    Option 3, initial δ^*, R_{δ^*}, and $(U_x \delta^*/U)$ are known.

3	ID	

DT(1)	RDT(1)	P	

X(1)	U(1)	1.0	RW(1)	VW(1)	SW(1)	CW(1)

THE INITIAL PROFILE IS TO BE CALCULATED USING THE INPUT PROFILE AS A ROUGH GUESS.

    Laminar Flow
        Option 4, $R_L = (x_2 - x_1)U(x_2)/\nu$ and B in equation (20) are known.
        (In this case $U^2(x_1)\ \delta^*(x_1) = 0$.)

4	ID	

0.0	RL	B	

X(1)	U(1)	0.0	RW(1)	0.0	0.0	CW(1)

X(2)	U(2)	0.0	RW(2)	VW(2)	0.0	CW(2)

- 47 -

<u>Option 5</u>, initial δ^*, $R_{\delta*}$, and $U_x\delta^{*2}/\nu$ are known.

5	10	

DT(1)	RDT(1)	P	

X(1)	U(1)	0.0	RW(1)	VW(1)	0.0	CW(1)

Turbulent Flow
   <u>Option 6</u>, initial δ^*, $R_{\delta*}$, and β are known.

6	10	

DT(1)	RDT(1)	B	

X(1)	U(1)	1.0	RW(1)	VW(1)	SW(1)	CW(1)

<u>Option 7</u>, initial δ^*, $R_{\delta*}$, and $(U_x\delta^*/U)$ are known.

7	10	

DT(1)	RDT(1)	P	

X(1)	U(1)	1.0	RW(1)	VW(1)	SW(1)	CW(1)

Only in <u>Option 4</u> is the calculation actually starting from the beginning of the boundary layer growth. In this case, the position X(1) corresponds to this initial point where either DT or U should be zero and the similarity laws are used to provide values at X(2). Therefore, to calculate a plane stagnation point flow (B = 1.0), for example, U(1) must be 0.0. Similarly,

a Blasius flow (B = 0.0, DT(1) must be 0.0. RW(1) may be either finite or zero depending on the geometry. For the flow starting at the apex of a cone, RW(1) will be zero but RW(2) will be nonzero. On the other hand, a flat plate flow will have all RW values equal to zero, as described below. Also, in <u>Option 4</u>, VW(1) must be zero since transpiration is not compatible with the boundary layer assumptions for $U^2\delta^* = 0$.

Having initialized the calculation, it remains to specify the information required for the downstream calculations. This is accomplished with a set of cards each containing the parameters related to an x-station. The first two parameters, X and U must appear on each card. The remainder need not be specified unless they apply in a particular case.

X(1)	U(1)	TURB(1)	RW(1)	VW(1)	SW(1)	CW(1)

X(I+1)	U(I+1)	TURB(I+1)	RW(I+1)	VW(I+1)	SW(I+1)	CW(I+1)

-10000.0	

The sequence of X values on successive cards define the x spacing at which calculations will be performed. As in the case of the η spacing, there is no mechanism for altering the x step size to maintain accuracy. The reason for this is that there is generally no need for it. The numerical method itself is sufficiently forgiving to be accurate over a wide range of step size. The controlling consideration then is to represent the mainstream velocity distribution realistically. But since this is known, the x steps can be chosen in advance. In the case of layers near equilibrium, steps of the order of many hundreds of displacement thicknesses are possible. On the other hand, if the mainstream velocity changes rapidly, steps may be small.

The values of U, corresponding to each X, define the mainstream velocity. U may be in either dimensional or nondimensional form since it appears only as a ratio in the calculations.

The third quantity that can be specified is TURB, which indicates whether the flow is laminar (TURB = 0.0) or turbulent (TURB = 1.0). If the flow is laminar no empirical content is necessary since the laminar boundary layer equations are complete. However, if the flow is turbulent a semi-empirical effective viscosity assumption is necessary to close the equations. The form of this assumption is given in Section I and the basis for the assumption is discussed in greater detail in Reference [1]. TURB also has another function. By changing TURB from 0.0 to 1.0, either abruptly or gradually over the distance of several x stations, the effect of transition can be simulated. There is no mechanism within the program to decide when or how this should be done. This information must be supplied by the user, either from consideration of the boundary conditions or from previous calculation attempts (see, for instance, [11], Chaps. XVI and XVII).

RW is the radius of the wall in a flow over an axisymmetric body. The units of RW must be the same as those of X. For a planar flow RW may be left blank, which is treated as an infinite radius of curvature.

It is possible to calculate flows with transpiration or aspiration by specifying VW = $v_w(x)$. VW is positive for transpiration and negative for aspiration.

The last boundary condition that can be specified is the wall roughness. This is done in the form of an average roughness size, SW = s_w. Again the units of SW must be the same as those of X.

In preparation for future implementation, provisions have also been made for input and storage of the longitudinal radius of curvature, CW = c_w.

Finally, there are several constants which are not data inputs but are set permanently within the program to values which suit the particular computer to be used. The first two of these are JD and ID, which specify the maximum number of calculation points perpendicular to and parallel to the wall respectively. The values given in the listing (JD = 250, ID = 60) are considerably larger than necessary in most practical calculations. It is possible that in some cases they may need to be made larger or smaller depending on the storage capacity of the computer to be used. This may be done by changing the values in the data statement and also by specifying consistent values in the dimension statements both of which are at the beginning of the main program.

The other number which must be set is SYG in $PROFYL. SYG should be set equal to 10^{n-3} where n is the nearest whole number of significant figures of accuracy which the computer carries during computations. For example, if the computer performs calculations to seven and a half significant figures, n = 7.

Output

The primary output from the calculations is a list of the calculated profiles and parameters that is printed out with the subroutine $FILE for each x station (see, for example, Table 1c. This output form gives the principle input and output parameters at the top. Below these are the various profiles as functions of J and y/δ*. The profiles are identified with symbols which are for the most part identical with variable names used in the program (see notation). The exceptions are YY+, U+, and YY*SF which represent the law of the wall coordinates yu_τ/ν and u/u_τ and y/θ, respectively. The three columns labeled VH, VHP, and VHPP are not used but are included for the convenience of the user so that other profiles may be printed out.

The output of the calculation begins with a print-out of the input profiles and other related profiles which have been calculated from it with $FILE. The next page begins with a list of the input parameters for reference. These are identified and are, therefore, self-explanatory. Below this, in the case that the initial profile is recalculated, is a list of significant parameters for each iteration indicating, among other things, the rate of convergence. If the initial profile is not to be recalculated, only one line appears which gives the parameters calculated from the input data. Here, as in the output of $FILE, the parameters are identified with the same symbols as were used in the program. The first five are indices of key points on the profile which are shown in Figure 4 in Section IV. A line which begins with JK designates a completed primary iteration with all of the pertinent parameters which were involved. Occasionally it is necessary to perform secondary iterations in $PROFYL, as described in Section III, in order to obtain a valid profile for each primary iteration. If such secondary iterations are performed, they are shown on lines beginning with JS above the primary iteration line.

The next five are parameters relating to the profile as a whole. FJE is F(JE), the integral

$$f(\infty) = \int_0^\infty f'(\eta)d\eta \; ,$$

and FPPW is f"(o). The remaining numbers are concerned with the asymptotic outer solution and the outer boundary condition. Finally the recalculated profile is printed out with $FILE and this signifies the end of the initialization.

The output of the calculation moving downstream consists of two parts. The first is the list of significant parameters for each iteration as in the case of the recalculated profile. This time, however, an indication of the accuracy of the numerical integration

along the wall is printed out. This is done in the form of two numbers, one on either side of an equal sign which correspond to the left- and right-hand sides of equation (18) in Section II. Closer agreement between the two numbers indicates more accurate integration.

The second part of the output at each x station is the print out of the profiles and parameters with $FILE.

Finally, at the end of the calculation for the entire series of x stations, a summary of the important integral parameters of the flow is printed out for convenience.

Illustrative Examples

Calculations have been performed for three boundary layers which illustrate some of the capabilities of the program.

The first example is the classical case of the Howarth flow, which is described in Reference [11], p. 156. This flow is entirely laminar with a free stream velocity distribution given by the relation

$$\underset{\sim}{U} = \underset{\sim}{U}_o - a\underset{\sim}{x},$$

which produces nonsimilar velocity profiles.

In the calculation $U(X)$ is taken to be $\underset{\sim}{U}/\underset{\sim}{U}_o$, $X = \underset{\sim}{x}$ and $a/\underset{\sim}{U}_o = 10^{-3}$ for convenience. The calculation begins by computing a Blasius starting profile at $X = 0$. The layer then grows in an adverse pressure gradient until it separates at $X = 125$. A listing of the necessary input cards is given in Table 1a.

A sample listing of the profiles for $X = 100$ is shown in Table 1c preceded by a print out of the iterations which produced it.

Finally, the summary of the integral parameters is given in Table 1d. Perhaps the most interesting feature is the behavior of the skin friction coefficient as separation is approached. C_f is plotted in Figure 5 and is shown to extrapolate to zero at $X = 120$ as expected. More calculations could have been made in the neighborhood of separation (and, therefore, the separation point could have been defined more precisely) if closer η spacing had been used. This restriction was explained in the section on inputs.

HOWARTH'S FLOW

```
  8
 30
0.0        0.2        0.4        0.6        0.8        1.0
1.2        1.4        1.6        1.8        2.0        2.2
2.4        2.6        2.8        3.0        3.2        3.4
3.6        3.8        4.0        4.2        4.4        4.6
4.8        5.0        5.2        5.4        5.6        5.8
 24
1.0        0.885      0.770      0.657      0.547      0.444
0.349      0.265      0.193      0.135      0.0902     0.0648
0.0394     0.01832    0.00994    0.00575    0.00383    0.0019
0.00077    0.00045    0.00012    0.00007    0.00003    0.0
  4              0
0.0        10.0       0.0
0.0        1.0        0.0
2.0        0.998      0.0
5.0        0.995      0.0
10.0       0.990      0.0
15.0       0.985      0.0
20.0       0.980      0.0
30.0       0.970      0.0
40.0       0.960      0.0
50.0       0.950      0.0
60.0       0.940      0.0
70.0       0.930      0.0
80.0       0.920      0.0
90.0       0.910      0.0
100.0      0.900      0.0
110.0      0.890      0.0
120.0      0.880      0.0
-10000.0
```

Table 1a.   Howarth's Flow
Input Data

INPUT VARIABLES FOR   HOWARTH'S FLOW

X = 1.20E 02   U = 8.80E-01   TURB =0.0   BW = 0.0   VW = 0.0   SW = 0.0   CW = 0.0

VALUES OF IMPORTANT VARIABLES FOR EACH ITERATION

JS	JM	JA	JE	PJE	FPPW	DT	PM	QM	S	CAP	FPA	FPPA	FHPA	FHPPA
1	76	112	172		-2.03E-01					1.08E-01	-9.31E 02	-8.54E 03	8.66E 03	7.93E 04
	112	112	165	1.02E 00	-9.55E-02	1.42E 01	-1.51E-02	1.35E-01	2.88E-01	1.12E-06	-1.85E 01	-1.38E 02	1.66E 07	1.24E 08
1	86	112	165		-9.55E-02					2.18E-02	-9.26E 02	-7.68E 03	4.24E 04	3.52E 05
	112	112	162	1.02E 00	-7.37E-02	1.45E 01	-1.53E-02	1.68E-01	2.75E-01	-4.05E-07	2.29E 00	1.55E 01	5.66E 06	3.83E 07
1	95	112	162		-7.37E-02					6.42E-03	-9.73E 02	-6.99E 03	1.52E 05	1.09E 06
	112	112	159	1.01E 00	-6.73E-02	1.49E 01	-1.55E-02	2.02E-01	2.63E-01	-4.91E-06	1.25E 01	7.67E 01	2.55E 06	1.56E 07
1	103	112	159		-6.73E-02					1.83E-03	-9.16E 02	-5.91E 03	5.00E 05	3.23E 06
	112	112	158	1.00E 00	-6.55E-02	1.50E 01	-1.56E-02	2.14E-01	2.59E-01	1.52E-06	-3.05E 00	-1.80E 01	2.01E 06	1.19E 07
1	111	112	158		-6.55E-02					6.00E-04	-9.89E 02	-5.85E 03	1.65E 06	9.76E 06
	112	112	157	1.00E 00	-6.49E-02	1.50E 01	-1.56E-02	2.17E-01	2.58E-01	-2.57E-06	4.92E 00	2.88E 01	1.91E 06	1.12E 07
	108	112	158	1.00E 00	-6.46E-02	1.50E 01	-1.56E-02	2.17E-01	2.57E-01	2.25E-04	-2.33E 02	-1.42E 03	1.04E 06	6.32E 06

INTEGRAL OF MOMENTUM EQUATION   CUMULATIVE MOMENTUM INTEGRAL

1.0053 = 1.0057   0.99047

Table 1b. Howarth's Flow
Significant Parameters for Each Iteration at X = 120

BOUNDARY LAYER PROFILES FOR

HOWARTH'S FLOW

X = 120.000   U = 0.880   RDT = 6.63E 01

TURB =0.0   RW = 0.0   VW = 0.0   SW = 0.0   CW = 0.0

DT = 1.50E 01   MT = 4.13E 00   CF = 1.95E-03   SF = 3.64E 00

J	YY	U/U	YY+	U+	VE	TAU	F	FP	FPP	0.0	0.0	1.00E 00	YY*SF
1	0.0	0.0	0.0	0.0	1.51E-02	9.76E-04	0.0	1.00E 00	-6.46E-02	0.0	0.0	1.00E 00	0.0
9	2.00E-01	3.48E-02	4.14E-01	1.11E 00	1.51E-02	4.26E-03	1.97E-01	9.65E-01	-2.82E-01	2.71E-03	2.00E-01	1.01E 00	7.27E-01
17	4.00E-01	1.12E-01	8.28E-01	3.58E 00	1.51E-02	7.29E-03	3.83E-01	8.88E-01	-4.83E-01	1.56E-02	4.13E-01	1.16E 00	1.45E 00
25	6.00E-01	2.25E-01	1.24E 00	7.22E 00	1.51E-02	9.76E-03	5.50E-01	7.75E-01	-6.46E-01	4.31E-02	6.97E-01	1.84E 00	2.18E 00
33	8.00E-01	3.66E-01	1.66E 00	1.17E 01	1.51E-02	1.13E-02	6.91E-01	6.34E-01	-7.47E-01	8.42E-02	1.29E 00	4.84E 00	2.91E 00
41	1.00E 00	5.19E-01	2.07E 00	1.66E 01	1.51E-02	1.15E-02	8.03E-01	4.81E-01	-7.64E-01	1.33E-01	3.35E 00	1.96E 01	3.64E 00
49	1.20E 00	6.65E-01	2.48E 00	2.13E 01	1.51E-02	1.04E-02	8.84E-01	3.35E-01	-6.90E-01	1.81E-01	1.30E 01	9.87E 01	4.36E 00
57	1.40E 00	7.90E-01	2.90E 00	2.53E 01	1.51E-02	8.26E-03	9.38E-01	2.10E-01	-5.47E-01	2.20E-01	6.50E 01	5.43E 02	5.09E 00
65	1.60E 00	8.83E-01	3.31E 00	2.83E 01	1.51E-02	5.69E-03	9.70E-01	1.17E-01	-3.77E-01	2.47E-01	3.53E 02	3.00E 03	5.82E 00
73	1.80E 00	9.42E-01	3.73E 00	3.02E 01	1.51E-02	3.38E-03	9.87E-01	5.80E-02	-2.24E-01	2.63E-01	1.89E 03	1.57E 04	6.54E 00
81	2.00E 00	9.75E-01	4.14E 00	3.12E 01	1.51E-02	1.72E-03	9.95E-01	2.51E-02	-1.14E-01	2.70E-01	9.51E 03	7.46E 04	7.27E 00
89	2.20E 00	9.91E-01	4.55E 00	3.17E 01	1.51E-02	7.43E-04	9.99E-01	9.48E-03	-4.92E-02	2.73E-01	4.34E 04	3.18E 05	8.00E 00
97	2.40E 00	9.97E-01	4.97E 00	3.19E 01	1.51E-02	2.80E-04	1.00E 00	3.16E-03	-1.86E-02	2.75E-01	1.78E 05	1.20E 06	8.73E 00
105	2.60E 00	9.99E-01	5.38E 00	3.20E 01	1.51E-02	8.85E-05	1.00E 00	9.00E-04	-5.86E-03	2.75E-01	6.54E 05	4.10E 06	9.45E 00
113	2.80E 00	1.00E 00	5.80E 00	3.20E 01	1.51E-02	2.49E-05	1.00E 00	2.36E-04	-1.65E-03	2.75E-01	2.71E 06	1.78E 07	1.02E 01
121	3.00E 00	1.00E 00	6.21E 00	3.20E 01	1.51E-02	6.34E-06	1.00E 00	5.40E-05	-4.20E-04	2.75E-01	4.55E 06	2.89E 07	1.09E 01
129	3.20E 00	1.00E 00	6.62E 00	3.20E 01	1.51E-02	1.36E-06	1.00E 00	1.06E-05	-9.03E-05	2.75E-01	2.79E 05	1.38E 06	1.16E 01
137	3.40E 00	1.00E 00	7.04E 00	3.20E 01	1.51E-02	2.49E-07	1.00E 00	1.77E-06	-1.65E-05	2.75E-01	9.59E 02	2.42E 03	1.24E 01
145	3.60E 00	1.00E 00	7.45E 00	3.20E 01	1.51E-02	3.87E-08	1.00E 00	2.54E-07	-2.57E-06	2.75E-01	7.45E 28	3.53E 18	1.31E 01
153	3.80E 00	1.00E 00	7.87E 00	3.20E 01	1.51E-02	5.12E-09	1.00E 00	3.12E-08	-3.39E-07	2.75E-01	4.72E-79	3.37E-80	1.38E 01

Table 1c. Howarth's Flow
Output of Profiles by $FILE for X = 120

PRINCIPAL BOUNDARY LAYER PARAMETERS FOR
HOWARTH'S FLOW

X	DT	MT	SF	CF	RDT	U	TURB	RW	VW	SW	CW
0.0	0.0	0.0	2.5900	0.0	0.	1.0000	0.0	0.0	0.0	0.0	0.0
2.0000	1.0858	0.4192	2.5900	0.210961	5.	0.9980	0.0	0.0	0.0	0.0	0.0
5.0000	1.7540	0.6765	2.5925	0.128684	9.	0.9950	0.0	0.0	0.0	0.0	0.0
10.0000	2.5114	0.9633	2.6071	0.088482	12.	0.9900	0.0	0.0	0.0	0.0	0.0
15.0000	3.1186	1.1869	2.6274	0.071080	15.	0.9850	0.0	0.0	0.0	0.0	0.0
20.0000	3.6478	1.3808	2.6417	0.060137	18.	0.9800	0.0	0.0	0.0	0.0	0.0
30.0000	4.5961	1.7200	2.6721	0.046497	22.	0.9700	0.0	0.0	0.0	0.0	0.0
40.0000	5.4767	2.0211	2.7097	0.037859	26.	0.9600	0.0	0.0	0.0	0.0	0.0
50.0000	6.3317	2.3013	2.7514	0.031543	30.	0.9500	0.0	0.0	0.0	0.0	0.0
60.0000	7.1868	2.5691	2.7975	0.026474	34.	0.9400	0.0	0.0	0.0	0.0	0.0
70.0000	8.0705	2.8297	2.8521	0.022162	38.	0.9300	0.0	0.0	0.0	0.0	0.0
80.0000	9.0097	3.0869	2.9187	0.018291	42.	0.9200	0.0	0.0	0.0	0.0	0.0
90.0000	10.0320	3.3434	3.0005	0.014658	46.	0.9100	0.0	0.0	0.0	0.0	0.0
100.0000	11.1958	3.6015	3.1086	0.011063	50.	0.9000	0.0	0.0	0.0	0.0	0.0
110.0000	12.6944	3.8638	3.2855	0.007175	57.	0.8900	0.0	0.0	0.0	0.0	0.0
120.0000	15.0285	4.1332	3.6360	0.001952	66.	0.8800	0.0	0.0	0.0	0.0	0.0

Table 1d. Howarth's Flow
Summary of Important Input and Output
Parameters for Entire Calculation
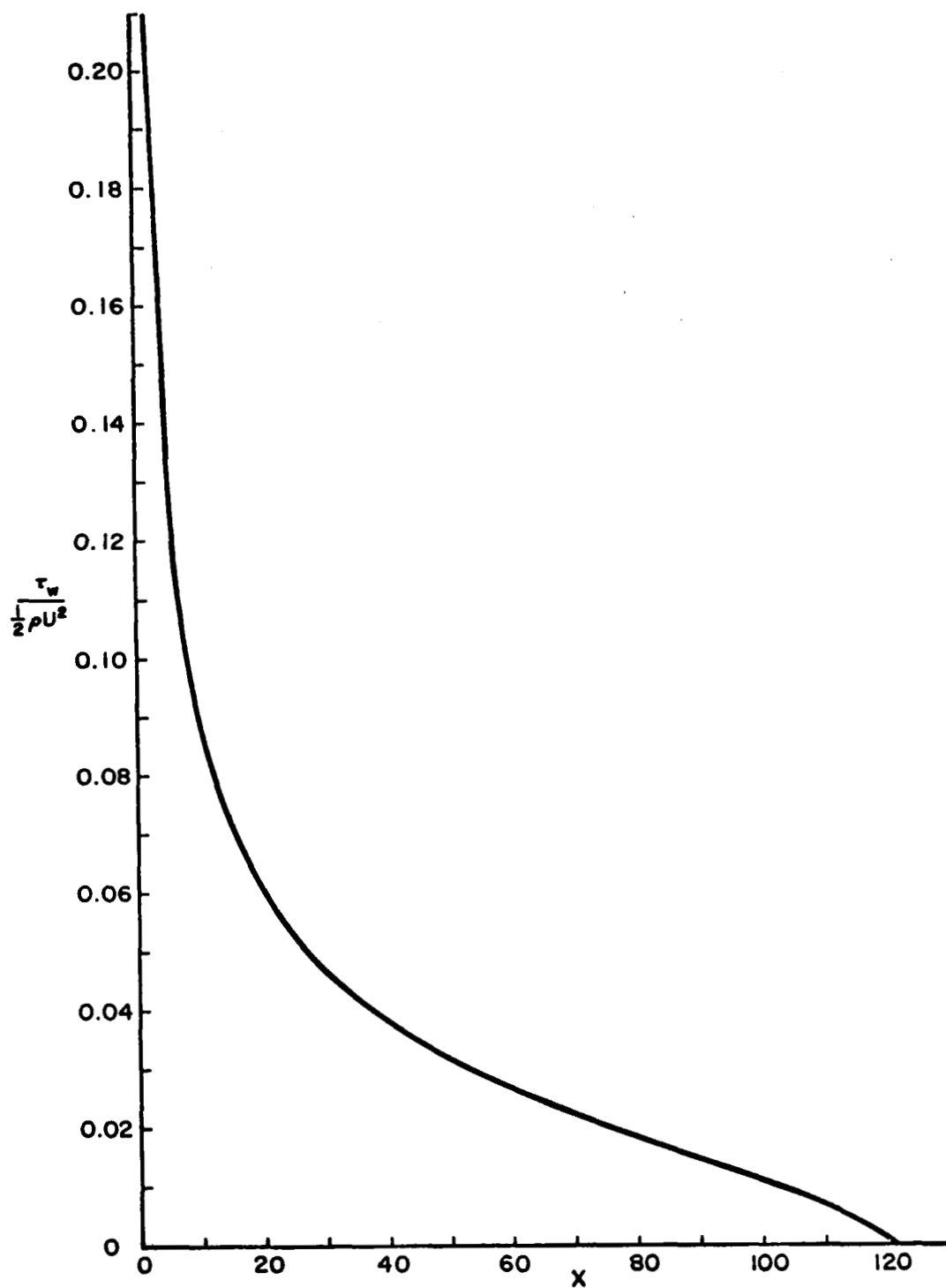
Figure 5. C_f vs. X for the Example Numerical Calculation
of Howarth's Flow.

The next example is that of an entirely turbulent flow. It is a prediction of some turbulent boundary layer data (case 6) measured by Moses [16] on an axisymmetric body, which was used at the Stanford Symposium on the "Computation of Turbulent Boundary Layers." This case provides an example of the accuracy of the method for turbulent flow.

Here the calculation begins with the experimental profile for the first station X = 0.0 (ft.) as an initial condition. The first portion of the flow is in an adverse pressure gradient. Then the pressure gradient is removed and the layer relaxes in conditions of nearly zero pressure gradient. This case provides an interesting variety of conditions.

The input listing is given in Table 2a. Lastly, the summary of integral parameters is given in Table 2c. Comparison with Moses' data is afforded by Figure 6 where C_f and H are plotted against the data points. The predictions are shown to be good.

The final example is of a more involved nature. It has been constructed specially to demonstrate various aspects of the capability of the program. Therefore, there are no data or analytical results against which to verify the results.

The flow pattern might be envisioned as the flow over the suction side of a turbine blade. The boundary layer begins with a laminar, similarity stagnation point solution at the leading edge. It flows in a strong favorable pressure gradient to X = 1.75 at which point the pressure gradient suddenly changes to a strong adverse one. Taking into account the Reynolds number, the adverse pressure gradient and the high turbulence level usually associated with flows in turbomachinery, transition is assumed to take place almost instantly at X = 1.75. Following this the turbulent boundary layer develops in a strong adverse pressure gradient until the end of the blade is reached at X = 3.30. Between X = 1.9 and X = 3.0, uniform transpiration, V_w = 0.443, takes place.

The inputs required appear in Table 3a. An intermediate profile is given for X = 3.0 in Table 3c and the summary of integral parameters is shown in Table 3d.

Identification of Malfunctions

The calculations of the examples above all proceeded smoothly. However, this may not always be the case. To aid in the diagnosis of problems that may be encountered, some of the more common difficulties are discussed here.

```
                           IDENT. # 4100   MOSES CASE 6
  3
 42
0.              0.00200      0.00500      0.01000      0.02000      0.05000
0.10000         0.20000      0.30000      0.40000      0.50000      0.60000
0.70000         0.80000      0.90000      1.00000      1.20000      1.40000
1.60000         1.80000      2.00000      2.50000      3.00000      3.50000
4.00000         4.50000      5.00000      5.50000      6.00000      6.50000
7.00000         7.50000      8.00000      8.50000      9.00000      9.50000
10.00000       11.00000     12.00000     13.00000     14.00000     15.00000
 32
1.00000         0.99565      0.98911      0.97823      0.95646      0.89087
0.78156         0.60497      0.48038      0.41872      0.37795      0.35845
0.33906         0.32210      0.30705      0.29631      0.27175      0.24812
0.22682         0.20829      0.18961      0.14675      0.13007      0.08016
0.05501         0.03453      0.02119      0.01121      0.00481      0.00091
-0.00020        0.
  3      1
0.00162    884.00000    -0.00037
0.          88.00000     1.00000      0.23958      0.          0.
0.16200     83.95000     1.00000      0.23958      0.          0.
0.32300     78.56000     1.00000      0.23958      0.          0.
0.64600     68.62000     1.00000      0.23958      0.          0.
0.86500     65.02000     1.00000      0.23958      0.          0.
1.05800     64.31000     1.00000      0.23958      0.          0.
1.30300     64.13000     1.00000      0.23958      0.          0.
1.51600     64.13000     1.00000      0.23958      0.          0.
1.73400     64.00000     1.00000      0.23958      0.          0.
1.97900     64.13000     1.00000      0.23958      0.          0.
2.19800     63.88000     1.00000      0.23958      0.          0.
2.43800     64.19000     1.00000      0.23958      0.          0.
2.68300     63.76000     1.00000      0.23958      0.          0.
-10000.0
```

Table 2a. Moses, Case 6
Input Data

INPUT VARIABLES FOR   IDENT. # 4100   MOSES CASE 6

X = 8.65E-01   U = 6.50E 01   TURB =1.000   RW = 2.40E-01   VW = 0.0   SW = 0.0   CW = 0.0

VALUES OF IMPORTANT VARIABLES FOR EACH ITERATION

JK	JS	JM	JA	JE	FJE	FPPW	DT	PM	QM	S	CAP	FPA	FPPA	FHPA	FHPPA
41		85	85	108	9.33E-01	-5.26E 00	9.34E-03	-1.78E-03	8.45E-03	1.28E 00	-1.38E 00	1.91E 02	2.03E 02	1.39E 02	1.47E 02
41		85	85	110	1.00E 00	-4.81E 00	8.71E-03	-1.73E-03	5.62E-03	1.37E 00	4.55E-01	-1.16E 02	-1.41E 02	2.55E 02	3.09E 02
41		85	85	110	1.00E 00	-4.79E 00	8.74E-03	-1.74E-03	5.76E-03	1.37E 00	1.77E-02	-4.28E 00	-5.17E 00	2.42E 02	2.92E 02

INTEGRAL OF MOMENTUM EQUATION   CUMULATIVE MOMENTUM INTEGRAL

1.0393 = 1.0599   1.07634

Table 2b. Moses, Case 6
Significant Parameters for Each Iteration at X = 0.865

BOUNDARY LAYER PROFILES FOR
IDENT. # 4100 MOSES CASE 6

X =   0.865   U = 65.020   RDT = 3.60E 03

TURB =1.000   RW = 2.40E-01   VW = 0.0   SW = 0.0   CW = 0.0

DT = 8.92E-03   MT = 5.81E-03   CF = 2.72E-03   SF = 1.53E 00

J	YY	U/U	YY+	U+	VE	TAU	F	PP	PPP			YY*SF
1	0.0	0.0	0.0	0.0	2.84E-04	1.36E-03	0.0	1.00E 00	-4.79E 00	0.0	0.0	1.00E 00	0.0
4	2.00E-03	9.58E-03	2.65E-01	2.60E-01	2.84E-04	1.36E-03	1.99E-03	9.90E-01	-4.80E 00	9.52E-06	2.00E-03	1.00E 00	3.07E-03
7	5.00E-03	2.40E-02	6.62E-01	6.51E-01	2.84E-04	1.36E-03	4.94E-03	9.76E-01	-4.81E 00	5.90E-05	5.00E-03	9.99E-01	7.67E-03
10	1.00E-02	4.81E-02	1.32E 00	1.30E 00	2.84E-04	1.37E-03	9.76E-03	9.52E-01	-4.83E 00	2.32E-04	1.00E-02	9.99E-01	1.53E-02
13	2.00E-02	9.65E-02	2.65E 00	2.62E 00	2.85E-04	1.38E-03	1.90E-02	9.03E-01	-4.85E 00	9.01E-04	2.00E-02	9.95E-01	3.07E-02
16	5.00E-02	2.37E-01	6.62E 00	6.43E 00	3.29E-04	1.42E-03	4.40E-02	7.63E-01	-4.31E 00	5.03E-03	4.85E-02	8.70E-01	7.66E-02
19	9.99E-02	3.84E-01	1.32E 01	1.04E 01	8.33E-04	1.48E-03	7.80E-02	6.16E-01	-1.78E 00	1.58E-02	7.87E-02	3.85E-01	1.53E-01
22	1.99E-01	4.76E-01	2.64E 01	1.29E 01	2.99E-03	1.59E-03	1.34E-01	5.24E-01	-5.29E-01	4.04E-02	1.02E-01	1.72E-01	3.06E-01
25	2.99E-01	5.17E-01	3.96E 01	1.40E 01	5.05E-03	1.68E-03	1.84E-01	4.83E-01	-3.29E-01	6.54E-02	1.18E-01	1.58E-01	4.58E-01
28	3.98E-01	5.46E-01	5.27E 01	1.48E 01	7.05E-03	1.78E-03	2.31E-01	4.54E-01	-2.49E-01	9.03E-02	1.34E-01	1.64E-01	6.10E-01
31	4.96E-01	5.68E-01	6.58E 01	1.54E 01	9.07E-03	1.89E-03	2.76E-01	4.32E-01	-2.04E-01	1.15E-01	1.51E-01	1.74E-01	7.61E-01
34	5.95E-01	5.87E-01	7.88E 01	1.59E 01	1.11E-02	1.98E-03	3.18E-01	4.13E-01	-1.74E-01	1.40E-01	1.69E-01	1.85E-01	9.12E-01
37	6.93E-01	6.04E-01	9.18E 01	1.64E 01	1.32E-02	2.07E-03	3.59E-01	3.96E-01	-1.53E-01	1.64E-01	1.88E-01	1.97E-01	1.06E 00
40	7.91E-01	6.18E-01	1.05E 02	1.68E 01	1.53E-02	2.15E-03	3.98E-01	3.82E-01	-1.36E-01	1.88E-01	2.09E-01	2.11E-01	1.21E 00
43	8.88E-01	6.32E-01	1.18E 02	1.71E 01	1.60E-02	2.23E-03	4.35E-01	3.68E-01	-1.35E-01	2.11E-01	2.31E-01	2.45E-01	1.36E 00
46	9.86E-01	6.45E-01	1.31E 02	1.75E 01	1.60E-02	2.29E-03	4.72E-01	3.55E-01	-1.38E-01	2.34E-01	2.58E-01	2.92E-01	1.51E 00
49	1.18E 00	6.74E-01	1.56E 02	1.83E 01	1.60E-02	2.37E-03	5.41E-01	3.26E-01	-1.42E-01	2.80E-01	3.28E-01	4.04E-01	1.81E 00
52	1.37E 00	7.02E-01	1.82E 02	1.91E 01	1.60E-02	2.41E-03	6.03E-01	2.98E-01	-1.44E-01	3.23E-01	4.23E-01	5.46E-01	2.11E 00
55	1.56E 00	7.31E-01	2.07E 02	1.98E 01	1.60E-02	2.37E-03	6.61E-01	2.69E-01	-1.43E-01	3.64E-01	5.51E-01	7.31E-01	2.40E 00
58	1.75E 00	7.60E-01	2.33E 02	2.06E 01	1.60E-02	2.28E-03	7.12E-01	2.40E-01	-1.39E-01	4.02E-01	7.23E-01	9.75E-01	2.69E 00
61	1.94E 00	7.87E-01	2.58E 02	2.14E 01	1.60E-02	1.97E-03	7.58E-01	2.13E-01	-1.33E-01	4.38E-01	9.52E-01	1.30E 00	2.98E 00
64	2.41E 00	8.50E-01	3.20E 02	2.31E 01	1.60E-02	1.97E-03	8.49E-01	1.50E-01	-1.13E-01	5.13E-01	1.92E 00	2.67E 00	3.70E 00
67	2.88E 00	9.02E-01	3.81E 02	2.45E 01	1.60E-02	1.55E-03	9.12E-01	9.82E-02	-8.76E-02	5.67E-01	3.92E 00	5.51E 00	4.42E 00
70	3.34E 00	9.40E-01	4.42E 02	2.55E 01	1.60E-02	1.10E-03	9.52E-01	6.05E-02	-6.14E-02	6.04E-01	8.03E 00	1.13E 01	5.12E 00
73	3.79E 00	9.65E-01	5.02E 02	2.62E 01	1.60E-02	7.33E-04	9.76E-01	3.46E-02	-4.02E-02	6.27E-01	1.64E 01	2.26E 01	5.82E 00
76	4.24E 00	9.82E-01	5.62E 02	2.66E 01	1.60E-02	4.42E-04	9.89E-01	1.83E-02	-2.39E-02	6.40E-01	3.31E 01	4.46E 01	6.51E 00
79	4.68E 00	9.91E-01	6.21E 02	2.69E 01	1.60E-02	2.47E-04	9.96E-01	8.97E-03	-1.32E-02	6.47E-01	6.56E 01	8.56E 01	7.19E 00
82	5.12E 00	9.96E-01	6.79E 02	2.70E 01	1.60E-02	1.28E-04	9.99E-01	3.97E-03	-6.76E-03	6.50E-01	1.27E 02	1.60E 02	7.86E 00
85	5.56E 00	9.99E-01	7.36E 02	2.71E 01	1.60E-02	6.32E-05	1.00E 00	1.46E-03	-3.28E-03	6.51E-01	2.42E 02	2.92E 02	8.53E 00
88	5.99E 00	1.00E 00	7.93E 02	2.71E 01	1.60E-02	2.25E-05	1.00E 00	4.33E-04	-1.04E-03	6.52E-01	4.19E 01	3.10E 01	9.19E 00
91	6.41E 00	1.00E 00	8.50E 02	2.71E 01	1.60E-02	5.96E-06	1.00E 00	1.19E-04	-3.02E-04	6.52E-01	1.17E 01	1.37E 01	9.84E 00
94	6.84E 00	1.00E 00	9.06E 02	2.71E 01	1.60E-02	1.63E-06	1.00E 00	3.08E-05	-8.14E-05	6.52E-01	1.18E 08	-2.77E-56	1.05E 01
97	7.26E 00	1.00E 00	9.61E 02	2.71E 01	1.60E-02	4.14E-07	1.00E 00	7.42E-06	-2.05E-05	6.52E-01	2.52E 08	2.48E 28	1.11E 01
100	7.67E 00	1.00E 00	1.02E 03	2.71E 01	1.60E-02	9.86E-08	1.00E 00	1.68E-06	-4.82E-06	6.52E-01	-2.23E-67	4.56E-71	1.18E 01
103	8.08E 00	1.00E 00	1.07E 03	2.71E 01	1.60E-02	2.21E-08	1.00E 00	3.60E-07	-1.06E-06	6.52E-01	5.46E-71	1.11E-80	1.24E 01
106	8.49E 00	1.00E 00	1.12E 03	2.71E 01	1.60E-02	4.66E-09	1.00E 00	7.27E-08	-2.22E-07	6.52E-01	1.11E-80	-1.44E-55	1.30E 01
109	8.89E 00	1.00E 00	1.18E 03	2.71E 01	1.60E-02	9.29E-10	1.00E 00	1.40E-08	-4.38E-08	6.52E-01	-1.67E-52	2.47E 18	1.36E 01

Table 2c. Moses, Case 6
Output of Profiles by \$FILE for x = 0.865.

PRINCIPAL BOUNDARY LAYER PARAMETERS FOR
IDENT. # 4100   MOSES CASE 6

X	DT	MT	SF	CF	RDT	U	TURB	RW	VW	SW	CW
0.0	0.0016	0.0011	1.5155	0.004921	884.	88.0000	1.000	0.2396	0.0	0.0	0.0
0.1620	0.0025	0.0016	1.5136	0.004056	1297.	83.9500	1.000	0.2396	0.0	0.0	0.0
0.3230	0.0037	0.0024	1.5477	0.003411	1820.	78.5600	1.000	0.2396	0.0	0.0	0.0
0.6460	0.0071	0.0045	1.5682	0.002718	3022.	68.6200	1.000	0.2396	0.0	0.0	0.0
0.8650	0.0089	0.0058	1.5341	0.002716	3596.	65.0200	1.000	0.2396	0.0	0.0	0.0
1.0580	0.0094	0.0063	1.4942	0.002878	3762.	64.3100	1.000	0.2396	0.0	0.0	0.0
1.3030	0.0098	0.0067	1.4479	0.003116	3882.	64.1300	1.000	0.2396	0.0	0.0	0.0
1.5160	0.0101	0.0071	1.4228	0.003199	4005.	64.1300	1.000	0.2396	0.0	0.0	0.0
1.7340	0.0105	0.0075	1.4052	0.003259	4172.	64.0000	1.000	0.2396	0.0	0.0	0.0
1.9790	0.0109	0.0078	1.3911	0.003269	4329.	64.1300	1.000	0.2396	0.0	0.0	0.0
2.1980	0.0115	0.0083	1.3815	0.003296	4539.	63.8800	1.000	0.2396	0.0	0.0	0.0
2.4380	0.0117	0.0085	1.3728	0.003277	4670.	64.1900	1.000	0.2396	0.0	0.0	0.0
2.6830	0.0125	0.0092	1.3663	0.003294	4945.	63.7600	1.000	0.2396	0.0	0.0	0.0

Table 2d. Moses, Case 6
Summary of Important Input and Output
Parameters for Entire Calculation
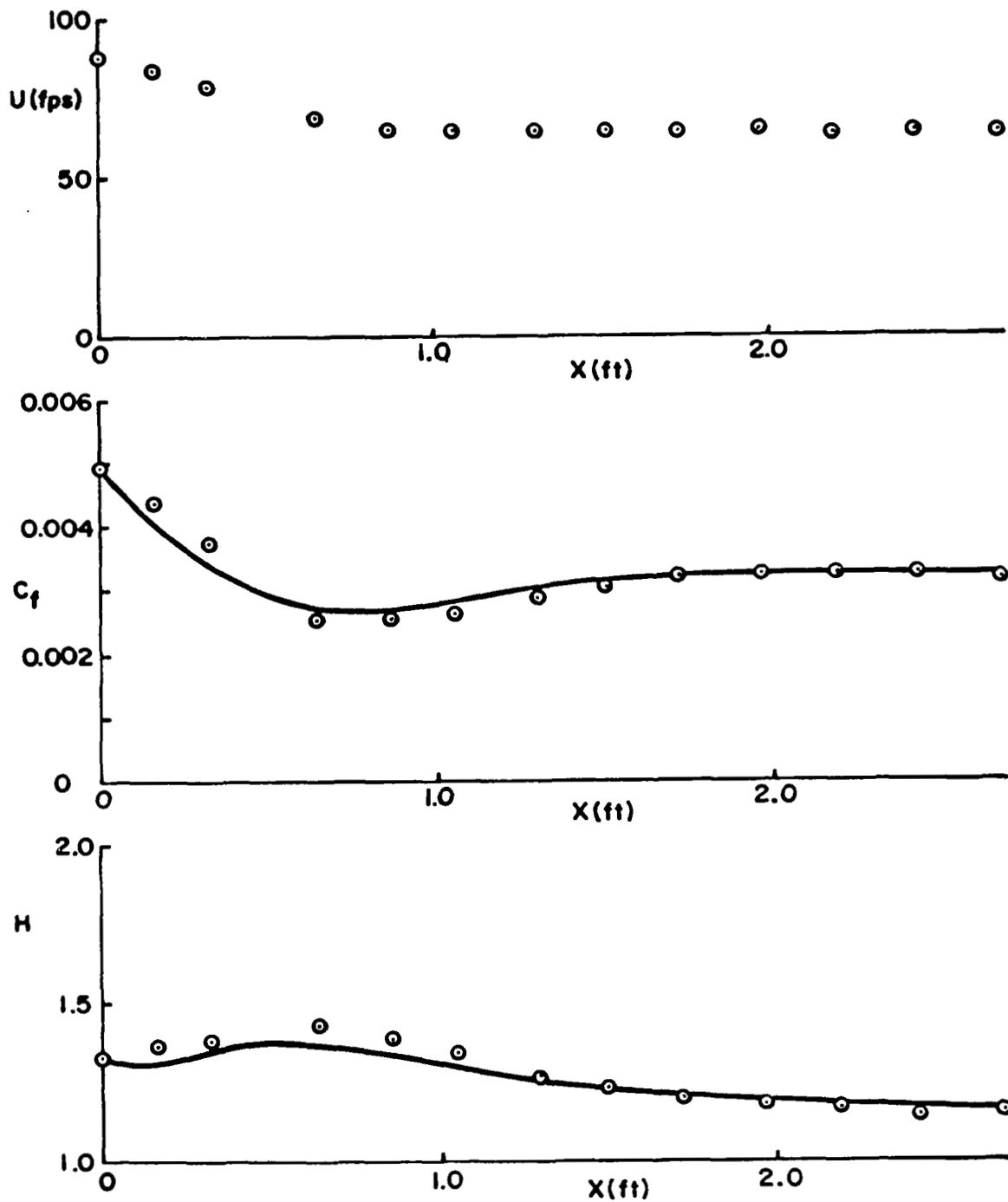
Figure 6. The example numerical prediction of C_f and H, shown with an unbroken line, compared with the values measured by Moses [16] for Case 6, which are indicated as open circles.

```
  3
 49
0.0        0.002      0.005      0.01       0.02       0.05
0.1        0.2        0.3        0.4        0.5        0.6
0.7        0.8        0.9        1.0        1.2        1.4
1.6        1.8        2.0        2.5        3.0        3.5
4.0        4.5        5.0        5.5        6.0        6.5
7.0        7.5        8.0        8.5        9.0        9.5
10.0       11.0       12.0       13.0       14.0       16.0
18.0       20.0       22.0       24.0       26.0       28.0
30.0
 30
1.0        0.998      0.996      0.992      0.985      0.962
0.925      0.853      0.785      0.721      0.660      0.603
0.549      0.498      0.451      0.407      0.329      0.263
0.207      0.161      0.123      0.0597     0.0267     0.0113
0.00495    0.00207    0.00021    0.00076    0.00024    0.0
  4      0
0.0        8000.0     1.0
0.0        0.0        0.0
0.100      56.8       0.0
0.110      60.4       0.0
0.125      65.2       0.0
0.150      74.0       0.0
0.175      78.4       0.0
0.200      88.8       0.0
0.225      95.2       0.0
0.250      101.2      0.0
0.300      111.2      0.0
0.350      122.0      0.0
0.400      131.2      0.0
0.450      138.8      0.0
0.500      146.0      0.0
0.600      158.4      0.0
0.700      168.4      0.0
0.800      178.0      0.0
0.900      186.0      0.0
1.000      199.2      0.0
1.100      217.6      0.0
1.200      242.0      0.0
1.300      270.0      0.0
1.400      294.0      0.0
1.500      314.0      0.0
1.600      331.2      0.0
1.700      336.8      0.0
1.750      335.6      1.0
1.800      328.0      1.0
1.850      310.8      1.0
1.900      293.6      1.0                    0.443
2.000      276.8      1.0                    0.443
2.100      264.8      1.0                    0.443
2.200      254.8      1.0                    0.443
2.300      246.0      1.0                    0.443
2.400      238.8      1.0                    0.443
2.500      234.0      1.0                    0.443
2.600      231.6      1.0                    0.443
2.700      227.6      1.0                    0.443
2.800      224.8      1.0                    0.443
2.900      223.6      1.0                    0.443
3.000      221.6      1.0                    0.443
3.100      218.8      1.0
3.200      216.4      1.0
3.300      214.0      1.0
-10000.0
```

Table 3a.   Suction Side of Turbine Blade
Input Data

INPUT VARIABLES FOR   THE SUCTION SIDE OF THE BLADE

X = 3.00E 00 U = 2.22E 02 TURB =1.000 RW = 0.0   VW = 4.43E-01 SW = 0.0   CW = 0.0

VALUES OF IMPORTANT VARIABLES FOR EACH ITERATION

JK	JS	JM	JA	JE	FJE	FPPW	DT	PM	QM	S	CAP	FPA	FPPA	FHPA	FHPPA
		70	81	102		-3.41E 00					8.68E-02	-7.28E 02	-4.72E 03	8.39E 03	3.14E 04
39	1	81	81	104	1.02E 00	-3.32E 00	1.41E-02	-1.25E-03	4.46E-03	1.91E 00	-1.16E-06	8.79E 00	3.20E 01	7.57E 06	2.76E 07
		71	81	104		-3.32E 00					-7.97E-02	8.87E 02	3.13E 03	1.11E 04	3.93E 04
39	1	81	81	103	1.00E 00	-3.40E 00	1.44E-02	-1.27E-03	7.70E-03	1.83E 00	1.74E-06	-5.94E 00	-1.97E 01	3.42E 06	1.13E 07
		73	81	103		-3.40E 00					-1.61E-02	5.87E 02	2.06E 03	3.65E 04	1.28E 05
39	1	81	81	103	1.00E 00	-3.42E 00	1.44E-02	-1.27E-03	7.70E-03	1.83E 00	-7.73E-07	2.69E 00	8.90E 00	3.48E 06	1.15E 07

INTEGRAL OF MOMENTUM EQUATION   CUMULATIVE MOMENTUM INTEGRAL

1.0356 = 1.0330   0.91837

Table 3b. Suction Side of Turbine Blade
Significant Parameters for Each Iteration at X = 3.0

BOUNDARY LAYER PROFILES FOR
THE SUCTION SIDE OF THE BLADE

X =   3.000   U = 221.600   RDT = 4.50E 03

TURB =1.000   RW = 0.0   VW = 4.43E-01   SW = 0.0   CW = 0.0

DT = 1.44E-02   MT = 8.77E-03   CF = 1.52E-03   SF = 1.64E 00

J	YY	U/U	YY+	U+	VE	TAU	P	FP	PPP				YY*SF
1	0.0	0.0	0.0	0.0	2.22E-04	7.58E-04	0.0	1.00E 00	-3.42E 00	0.0	0.0	1.00E 00	0.0
4	2.00E-03	6.91E-03	2.48E-01	2.51E-01	2.22E-04	7.75E-04	1.99E-03	9.93E-01	-3.49E 00	6.85E-06	2.02E-03	1.00E 00	0.0
7	5.00E-03	1.76E-02	6.19E-01	6.38E-01	2.22E-04	8.01E-04	4.96E-03	9.82E-01	-3.61E 00	4.30E-05	5.11E-03	1.05E 00	8.21E-03
10	1.00E-02	3.61E-02	1.24E 00	1.31E 00	2.22E-04	8.46E-04	9.82E-03	9.64E-01	-3.81E 00	1.73E-04	1.05E-02	1.09E 00	1.64E-02
13	2.00E-02	7.63E-02	2.48E 00	2.77E 00	2.23E-04	9.42E-04	1.93E-02	9.24E-01	-4.22E 00	6.99E-04	2.19E-02	1.20E 00	3.28E-02
16	5.00E-02	2.11E-01	6.19E 00	7.66E 00	2.91E-04	1.25E-03	4.50E-02	7.89E-01	-4.31E 00	4.33E-03	6.02E-02	1.25E 00	8.21E-02
19	1.00E-01	3.41E-01	1.24E 01	1.24E 01	1.13E-03	1.61E-03	8.06E-02	6.59E-01	-1.43E 00	1.45E-02	1.03E-01	5.92E-01	1.64E-01
22	2.00E-01	4.23E-01	2.48E 01	1.54E 01	3.54E-03	1.91E-03	1.42E-01	5.77E-01	-5.39E-01	3.81E-02	1.54E-01	5.33E-01	3.28E-01
25	3.00E-01	4.67E-01	3.71E 01	1.69E 01	5.68E-03	2.06E-03	1.97E-01	5.33E-01	-3.62E-01	6.28E-02	2.15E-01	6.85E-01	4.93E-01
28	4.00E-01	4.98E-01	4.95E 01	1.81E 01	7.85E-03	2.20E-03	2.49E-01	5.02E-01	-2.81E-01	8.78E-02	2.93E-01	8.80E-01	6.57E-01
31	5.00E-01	5.24E-01	6.19E 01	1.90E 01	1.01E-02	2.33E-03	2.98E-01	4.76E-01	-2.32E-01	1.13E-01	3.92E-01	1.11E 00	8.21E-01
34	6.00E-01	5.45E-01	7.43E 01	1.98E 01	1.24E-02	2.45E-03	3.44E-01	4.55E-01	-1.98E-01	1.38E-01	5.17E-01	1.39E 00	9.85E-01
37	7.00E-01	5.64E-01	8.66E 01	2.05E 01	1.47E-02	2.54E-03	3.89E-01	4.36E-01	-1.74E-01	1.62E-01	6.73E-01	1.73E 00	1.15E 00
40	8.00E-01	5.80E-01	9.90E 01	2.11E 01	1.60E-02	2.62E-03	4.32E-01	4.20E-01	-1.64E-01	1.87E-01	8.68E-01	2.26E 00	1.31E 00
43	9.00E-01	5.97E-01	1.11E 02	2.17E 01	1.60E-02	2.69E-03	4.73E-01	4.03E-01	-1.68E-01	2.11E-01	1.14E 00	3.16E 00	1.48E 00
46	1.00E 00	6.14E-01	1.24E 02	2.23E 01	1.60E-02	2.76E-03	5.12E-01	3.86E-01	-1.72E-01	2.35E-01	1.51E 00	4.37E 00	1.64E 00
49	1.20E 00	6.49E-01	1.49E 02	2.36E 01	1.60E-02	2.85E-03	5.86E-01	3.51E-01	-1.78E-01	2.81E-01	2.73E 00	8.26E 00	1.97E 00
52	1.40E 00	6.85E-01	1.73E 02	2.49E 01	1.60E-02	2.89E-03	6.53E-01	3.15E-01	-1.80E-01	3.26E-01	5.05E 00	1.57E 01	2.30E 00
55	1.60E 00	7.21E-01	1.98E 02	2.62E 01	1.60E-02	2.86E-03	7.12E-01	2.79E-01	-1.79E-01	3.68E-01	9.46E 00	3.01E 01	2.63E 00
58	1.80E 00	7.56E-01	2.23E 02	2.75E 01	1.60E-02	2.78E-03	7.64E-01	2.44E-01	-1.74E-01	4.06E-01	1.80E 01	5.85E 01	2.96E 00
61	2.00E 00	7.90E-01	2.48E 02	2.87E 01	1.60E-02	2.64E-03	8.10E-01	2.10E-01	-1.65E-01	4.41E-01	3.47E 01	1.15E 02	3.28E 00
64	2.50E 00	8.65E-01	3.09E 02	3.14E 01	1.60E-02	2.13E-03	8.95E-01	1.35E-01	-1.33E-01	5.12E-01	1.88E 02	6.48E 02	4.11E 00
67	3.00E 00	9.22E-01	3.71E 02	3.35E 01	1.60E-02	1.51E-03	9.47E-01	7.74E-02	-9.41E-02	5.59E-01	1.07E 03	3.78E 03	4.93E 00
70	3.50E 00	9.60E-01	4.33E 02	3.49E 01	1.60E-02	9.27E-04	9.76E-01	4.01E-02	-5.79E-02	5.86E-01	6.27E 03	2.22E 04	5.75E 00
73	4.00E 00	9.82E-01	4.95E 02	3.57E 01	1.60E-02	4.95E-04	9.90E-01	1.84E-02	-3.10E-02	5.99E-01	3.65E 04	1.28E 05	6.57E 00
76	4.50E 00	9.93E-01	5.57E 02	3.60E 01	1.60E-02	2.30E-04	9.96E-01	7.46E-03	-1.44E-02	6.05E-01	2.08E 05	7.17E 05	7.39E 00
79	5.00E 00	9.97E-01	6.19E 02	3.62E 01	1.60E-02	9.40E-05	9.99E-01	2.67E-03	-5.87E-03	6.08E-01	1.14E 06	3.85E 06	8.21E 00
82	5.50E 00	9.99E-01	6.81E 02	3.63E 01	1.60E-02	3.26E-05	9.99E-01	8.30E-04	-2.04E-03	6.09E-01	5.03E 02	6.49E 02	9.03E 00
85	6.00E 00	1.00E 00	7.43E 02	3.63E 01	1.60E-02	9.91E-06	1.00E 00	2.27E-04	-6.19E-04	6.09E-01	4.02E 02	2.52E 08	9.85E 00
88	6.50E 00	1.00E 00	8.04E 02	3.63E 01	1.60E-02	2.60E-06	1.00E 00	5.41E-05	-1.62E-04	6.09E-01	3.61E 02	2.52E 08	1.07E 01
91	7.00E 00	1.00E 00	8.66E 02	3.63E 01	1.60E-02	5.90E-07	1.00E 00	1.12E-05	-3.69E-05	6.09E-01	4.44E 12	2.52E 08	1.07E 01
94	7.50E 00	1.00E 00	9.28E 02	3.63E 01	1.60E-02	1.16E-07	1.00E 00	2.04E-06	-7.25E-06	6.09E-01	1.17E 01	1.37E 01	1.15E 01
97	8.00E 00	1.00E 00	9.90E 02	3.63E 01	1.60E-02	1.98E-08	1.00E 00	3.23E-07	-1.24E-06	6.09E-01	1.18E 08	-2.77E-56	1.23E 01
100	8.50E 00	1.00E 00	1.05E 03	3.63E 01	1.60E-02	2.92E-09	1.00E 00	4.46E-08	-1.83E-07	6.09E-01	-2.52E 08	2.48E 28	1.31E 01
103	9.00E 00	1.00E 00	1.11E 03	3.63E 01	1.60E-02	0.0	1.00E 00	0.0	0.0	6.09E-01	-2.23E-67	4.56E-71	1.40E 01
										6.09E-01	5.46E-71	1.11E-80	1.48E 01

Table 3c. Suction Side of Turbine Blade
Output of Profiles by $FILE for X = 3.0

PRINCIPAL BOUNDARY LAYER PARAMETERS FOR THE SUCTION SIDE OF THE BLADE

X	DT	MT	SF	CF	RDT	U	TURB	RW	VW	SW	CW
0.0	0.0007	0.0003	2.2145	0.0	0.	0.0	0.0	0.0	0.0	0.0	0.0
0.1000	0.0007	0.0003	2.2145	0.027504	58.	56.8000	0.0	0.0	0.0	0.0	0.0
0.1100	0.0008	0.0004	2.2687	0.022603	71.	60.4000	0.0	0.0	0.0	0.0	0.0
0.1250	0.0009	0.0004	2.2729	0.018502	83.	65.2000	0.0	0.0	0.0	0.0	0.0
0.1500	0.0009	0.0004	2.2572	0.015410	91.	74.0000	0.0	0.0	0.0	0.0	0.0
0.1750	0.0011	0.0005	2.2738	0.014710	122.	78.4000	0.0	0.0	0.0	0.0	0.0
0.2000	0.0009	0.0004	2.1941	0.012715	112.	88.8000	0.0	0.0	0.0	0.0	0.0
0.2250	0.0010	0.0004	2.2510	0.012227	134.	95.2000	0.0	0.0	0.0	0.0	0.0
0.2500	0.0011	0.0005	2.2706	0.009958	150.	101.2000	0.0	0.0	0.0	0.0	0.0
0.3000	0.0011	0.0005	2.2501	0.008925	179.	111.2000	0.0	0.0	0.0	0.0	0.0
0.3500	0.0012	0.0005	2.2504	0.007617	198.	122.0000	0.0	0.0	0.0	0.0	0.0
0.4000	0.0012	0.0005	2.2578	0.006932	222.	131.2000	0.0	0.0	0.0	0.0	0.0
0.4500	0.0013	0.0006	2.2799	0.006038	251.	138.8000	0.0	0.0	0.0	0.0	0.0
0.5000	0.0013	0.0006	2.2697	0.005554	272.	146.0000	0.0	0.0	0.0	0.0	0.0
0.6000	0.0014	0.0006	2.2871	0.004667	320.	158.4000	0.0	0.0	0.0	0.0	0.0
0.7000	0.0015	0.0007	2.2867	0.004126	367.	168.4000	0.0	0.0	0.0	0.0	0.0
0.8000	0.0016	0.0007	2.2871	0.003653	405.	178.0000	0.0	0.0	0.0	0.0	0.0
C.9000	0.0017	0.0008	2.2561	0.003630	446.	186.0000	0.0	0.0	0.0	0.0	0.0
1.0000	0.0016	0.0007	2.1776	0.003780	443.	199.2000	0.0	0.0	0.0	0.0	0.0
1.1000	0.0014	0.0007	2.1200	0.004085	433.	217.6000	0.0	0.0	0.0	0.0	0.0
1.2000	0.0012	0.0006	2.1031	0.004094	426.	242.0000	0.0	0.0	0.0	0.0	0.0
1.3000	0.0011	0.0005	2.1233	0.003848	433.	270.0000	0.0	0.0	0.0	0.0	0.0
1.4000	0.0012	0.0005	2.1807	0.003392	478.	294.0000	0.0	0.0	0.0	0.0	0.0
1.5000	0.0012	0.0005	2.2152	0.002970	529.	314.0000	0.0	0.0	0.0	0.0	0.0
1.6000	0.0012	0.0006	2.2713	0.002451	583.	331.2000	0.0	0.0	0.0	0.0	0.0

Table 3d. Suction Side of Turbine Blade
Summary of Important Input and Output Parameters for Entire Calculation

1.7000	0.0015	0.0006	2.4083	0.001793	701.	336.7998	0.0	0.0	0.0	0.0	0.0
1.7500	0.0013	0.0007	1.8439	0.003974	606.	335.5999	1.000	0.0	0.0	0.0	0.0
1.8000	0.0015	0.0009	1.6845	0.004513	674.	328.0000	1.000	0.0	0.0	0.0	0.0
1.8500	0.0020	0.0012	1.7099	0.003576	875.	310.7998	1.000	0.0	0.0	0.0	0.0
1.9000	0.0027	0.0016	1.7362	0.002615	1129.	293.5999	1.000	0.0	0.443000	0.0	0.0
2.0000	0.0040	0.0023	1.7559	0.002080	1542.	276.7998	1.000	0.0	0.443000	0.0	0.0
2.1000	0.0051	0.0029	1.7355	0.001994	1904.	264.7998	1.000	0.0	0.443000	0.0	0.0
2.2000	0.0064	0.0037	1.7352	0.001765	2291.	254.8000	1.000	0.0	0.443000	0.0	0.0
2.3000	0.0078	0.0045	1.7332	0.001628	2686.	246.0000	1.000	0.0	0.443000	0.0	0.0
2.4000	0.0091	0.0053	1.7306	0.001530	3069.	238.8000	1.000	0.0	0.443000	0.0	0.0
2.5000	0.0102	0.0060	1.7097	0.001574	3359.	234.0000	1.000	0.0	0.443000	0.0	0.0
2.6000	0.0109	0.0065	1.6887	0.001565	3561.	231.6000	1.000	0.0	0.443000	0.0	0.0
2.7000	0.0121	0.0072	1.6800	0.001532	3863.	227.6000	1.000	0.0	0.443000	0.0	0.0
2.8000	0.0130	0.0078	1.6660	0.001545	4107.	224.8000	1.000	0.0	0.443000	0.0	0.0
2.9000	0.0135	0.0082	1.6480	0.001579	4264.	223.6000	1.000	0.0	0.443000	0.0	0.0
3.0000	0.0144	0.0088	1.6422	0.001516	4495.	221.6000	1.000	0.0	0.443000	0.0	0.0
3.1000	0.0152	0.0094	1.6231	0.002005	4691.	218.8000	1.000	0.0	0.0	0.0	0.0
3.2000	0.0158	0.0099	1.6036	0.002181	4822.	216.4000	1.000	0.0	0.0	0.0	0.0
3.3000	0.0165	0.0104	1.5937	0.002098	4985.	214.0000	1.000	0.0	0.0	0.0	0.0

Table 3d. (cont.)

The most common of these is simply that the solution does not converge. Indications that this has happened appear in the numbers printed out during the sequence of iterations. To begin with, FJE should be between 0.995 and 1.005. Also, FPPW should have changed by less than 1/2% between the last two iterations for satisfactory convergence. Lack of convergence may result from a number of causes some of which will be described below.

The other important pattern of calculation failure is unjustified separation. This can be recognized by the fact that FPPW becomes positive, indicating a negative wall shear stress, and/or FJE becomes negative. Unjustified separation frequently occurs when the boundary layer is near actual separation. If one iteration behaves as if the separation point has been overstepped, then it is difficult for the calculation to recover.

Both of these malfunctions usually can be traced either to an actual error in the input data or to a poor choice of the X step size. In the latter case, the step size is too large if the shape factor is changing by more than about 5%. On the other hand, if there are numbers in the JS column of the iteration list to indicate iterations within $PROFYL, then, if possible, the step size should be made larger for efficiency.

Other possible causes may be choices of boundary conditions which are incompatible with the assumptions used to derive the basic boundary layer equations. For example, the program will not calculate a boundary layer with a step change in free stream velocity or transpiration with v_w/U near 0.1. A radius of curvature in axial flow which is smaller than the boundary layer thickness may also give trouble, and very large roughness elements of the order of $s_w/\delta^* = .1$ will not work for obvious reasons.

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